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THE

PHYSICAL REVIEW.

TEMPERATURE MEASUREMENTS TO 1600° C.

By Arthur L. Day and E. T. Allen.

IN a recent study of the isomorphism and thermal properties of the feldspars[1] in the laboratory of the U. S. Geological Survey, the authors prepared certain apparatus for generating, regulating and measuring high temperatures, which has proved easy of manipulation and uncommonly exact and which may have a wider interest. Through the courtesy of President Kohlrausch and Professor Holborn, of the Reichsanstalt, we were also enabled to obtain unusually direct comparisons with the gas thermometer scale of that institution, and to establish this scale in a conveniently accessible form for use in this country.

It is not our purpose at this time to enter upon an extended discussion of methods or principles of temperature measurement in the upper ranges. This has been done elsewhere,[2] and the difficulties to be dealt with have also received considerable attention. We desire merely to describe the application of the Reichsanstalt high temperature scale with such detail as may be necessary to make it

[1] The entire investigation will appear in a few months as a professional paper of the U. S. Geological Survey.

[2] See Le Chatelier and Boudouard, "High Temperature Measurement." (Translated by Burgess.) Ludwig Holborn and Arthur L. Day, "Ueber das Luftthermometer bei hohen Temperaturen." Ann. d. Phys. u. Chem., 68, p. 817, 1899. (Translated, Am. Jour. Sci., 4, 8, p. 165, 1899.) Ann. d. Phys., 4, 2, p. 505, 1900. (Translated, Am. Jour. Sci., 4, 10, p. 171, 1900.)

By the same authors: "Uber die Thermoelectricität einiger Metalle." Sitzungsber. Berl. Akad., p. 691, 1899. (Translated, Am. Jour. Sci., 4, 8, p. 303, 1899.)

And many others.

serviceable and with sufficient data to enable measurements of the order of accuracy of the scale itself to be made without the trouble of the inconvenient official calibration and incidental loss of time.

In the ordinary routine of temperature measurement in the laboratory it is rarely necessary to refer back to the fundamental temperature scale — the errors at ordinary temperatures are for most purposes insignificant — but in pyrometry, temperature is almost without significance until some specific reference to fundamental measurements is made.

The general plan of the apparatus used in these determinations may be assumed to be fairly well known. It is the same in all essential particulars as that used by Holborn and Day [1] in establishing the high temperature scale with the gas thermometer at the Reichsanstalt. And yet it is plain that such a scale requires some care in the transplanting, particularly as the authors were without a gas thermometer and were therefore not in position to make direct comparisons with the gas scale.

The temperatures were measured with thermo-elements exclusively. We obtained from Dr. Heraeus (Hanau, Germany) a set of four elements cut successively from the same roll of wire, which, when joined together, proved to be identically alike in their readings over the range of temperature covered by the gas scale of the Reichsanstalt (250° to 1150° C.), within the limits of observation error. Through the courtesy of Professor Holborn these were taken to the Reichsanstalt and measured in the original melting-point furnace with the same elements in terms of which the gas thermometer scale had been expressed, and five careful comparisons made. These were the melting points of the pure metals, cadmium (in air), zinc (in air), antimony (reducing atmosphere), silver (reducing atmosphere) and copper (in air). A fortunate circumstance made it possible to send these carefully calibrated elements to Washington by messenger, which made it certain that they suffered nothing in transit.

The elements were then further compared in an electric furnace, which will be described below, and the melting points of the same group of metals again determined in our laboratory. The metals

[1] Ludwig Holborn and Arthur L. Day, Am. Jour. Sci., 4, 8, 165, 1899.

used, however, were from other sources than those which had served for the calibration at the Reichsanstalt. When this test was finished, we were able to assure ourselves that, although all the constants in the measuring apparatus — thermoelements, resistances, standard cells, metals, etc. — had been changed in the transfer from the Reichsanstalt to the Geological Survey at Washington, the aggregate error nowhere exceeded 1° over the entire range from 250° to 1150°. It will be remembered that 1° was about the accuracy which the standard gas thermometer showed at 1000°. Our thermoelectrical system is therefore now doubly established — (1) by direct comparison and (2) through an independent series of metal melting points — upon the gas thermometer scale of the Reichsanstalt within the limits of error of the latter, and can be verified at any time with the help of two of the elements which have been laid aside for this purpose, or by means of the melting points of the metals. The scale is therefore permanent.

As the introduction of the standard high temperature scale of the Reichsanstalt into this country and its establishment by proper fixed points may be a matter of considerable interest to other investigators, some further details regarding the metals chosen for these fixed points are added. We tried to find metals which should not only be of the purity necessary for such standards, but which should be easily obtainable in uniform quality. With four of the five metals of the Reichsanstalt series — cadmium, zinc, silver and copper — no difficulty was experienced, but we were not able to find satisfactory antimony in this country. This need not prove an obstacle, for the four points mentioned will serve most purposes without a fifth, while if the needs of an experiment are so exacting as to require an intermediate melting point, antimony can be imported from Kahlbaum of Berlin without great delay or excessive cost, in the same purity as that originally used at the Reichsanstalt.

The cadmium and zinc in our series were taken from the regular listed chemicals of Eimer and Amend (zinc, "C.P. in sticks"; "cadmium, metal sticks"), the silver was the well-known test silver of the Philadelphia Mint laboratory, while the copper was also from Eimer & Amend ("C.P. copper drops, cooled in hydrogen").

Careful analyses of samples of the cadmium, zinc and copper follow:

Zinc.		Cadmium.		Copper.	
Eimer & Amend's "c. p. in Sticks."		Eimer & Amend's in Sticks.		Eimer & Amend's "c. p. Drops, Cooled in Hydrogen."	
As	none.	As	none.	Te and Se	none.
Cu	trace.	Cu	trace.	Sb	trace?
Pb	.0412%	Pb	.0860%	As	none.
Cd	.0021	Zn	trace.	Bi	none.
Fe	.0053	Fe	.0025	Ag	.018%
Co and Ni	none.	Co and Ni	none.	Pb	.001
S	.005	S	.0005	Co and Ni	none.
	.0005			Zn	.010
				Fe	.011[1]
Total Impurities, .0491%		Total Impurities, .0890%		Total Impurities, .040%	

It was not deemed necessary to make an analysis of the silver as we were assured that it contained no impurity which could be quantitatively determined.

The melting temperatures of cadmium and zinc are relatively low and those of silver and copper comparatively high on the gas scale, with a long interval between, so that it sometimes becomes very desirable to have an intermediate point. The two melting points which are most conveniently located are aluminium and antimony. Aluminium, on account of its low density, and perhaps because it has been less successfully purified than the other metals, does not give a sharp and satisfactory melting point. The melting point of Kahlbaum's antimony, of which a recently published analysis is reproduced here, serves this purpose excellently. It rarely solidifies without considerable undercooling, but the point to which the temperature rises after crystallization begins is sensibly identical with the melting point.

Antimony (Kahlbaum, Berlin).[2]

Fe	.012%
Cu	.004
Pb	.003
	.019%

The C. P. antimony obtained from Eimer & Amend and from Merck & Co., in a careful analysis for which we are indebted to

[1] This figure is doubtless somewhat too high.

[2] Fritz Henz, Inaugural Dissertation, Zürich. Published Leipzig, 1903.

Dr. W. F. Hillebrand, of the Geological Survey, each showed about one per cent. of the sulphide still present and traces of other impurities. The melting temperatures of these varied under different conditions as much as fifteen degrees and were totally unsuited to this work.

Inasmuch as the melting points of these metals were determined with thermoelements which Professor Holborn had just calibrated with the metals in use at the Reichsanstalt for this purpose, a comparison of the values obtained will show the accuracy with which one may reproduce the Reichsanstalt scale entirely from local sources :

	Reichsanstalt.	Day & Allen.	Difference.
Cadmium.	321.7° (Kahlbaum)	321.7 (Eimer & Amend).	0.0°
Zinc.	419.0 (Kahlbaum)	420.0 (Eimer & Amend.)	1.0
Silver.	961.5 (Gold u. Silber Scheideanstalt).	962.2 (Philadelphia Mint).	0.7
Copper (in air).	1064.9 (Haddernheim Kupferwerk).	1065.3 (Eimer & Amend).	0.4
Copper (reducing atmosphere).	1084.1 (Haddernheim Kupferwerk).	1083.6 (1) (Eimer & Amend).	0.5

For the method of extrapolation of the scale and further information regarding the use and accuracy of thermo-elements at these temperatures, reference is made to the papers of Holborn and Day already cited.

For everyday use, four more elements were prepared and calibrated in the same way. Of these, two are of the usual form (Fig. 1) and two are of a new design (Fig. 2), which has proved very effective in the determination of the melting points of non-metallic substances. It will be seen from the diagram of the insulated element that the hot junction is protected from the melting charge by a casing of platiniridium (0.1 mm. thick) and by a protecting tube of refractory Berlin (Marquardt) porcelain (1.5 mm. thick). Very early in our experiments upon the mineral silicates we became

[1] A single determination with one element, all others are mean values with two or more elements.

aware that the conductivity of these materials for heat would be much poorer than in similar charges of metal. Furthermore, the charge of mineral which the furnaces could carry was only one fourth to one third as great as the metal charges used in the calibrations because of the great difference in specific gravity and the limited space which could be heated to a fairly uniform temperature. For these reasons the changes of state would be less sharply marked upon the heating and cooling curves than metal melting points, and it was feared that the readings of the protected element might prove too high or too low through inability to take on the temperature of the surrounding mass promptly. It was to discover and obviate this possible source of error that the form of thermo-element indicated in the adjoining diagram was devised. It really amounts to nothing more than the ordinary form of platinum — platin-rhodium element with the platinum wire insulated from the other by a very slender porcelain (Marquardt) tube and the platin-rhodium wire broadened out and wrapped around this tube like a cap over the portion which dips into the charge. The hot junction (*a*) is then the lower extremity of the cap where the platinum wire emerges from its insulating tube and is welded inside the platin-rhodium cap.

Fig. 1.

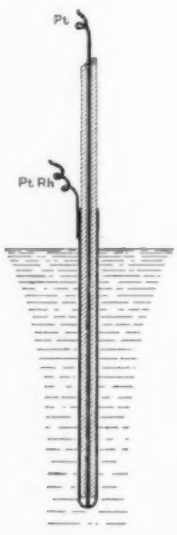

Fig. 2.

This form was dictated entirely by experience to meet conditions where an exposed element might be necessary or desirable. The wires of an ordinary element, if imbedded without protection, are rather frail for the wear and tear of breaking or drilling mineral charges out of the crucibles after the measurements, and they can not be strengthened without introducing a greater error through the

amount of heat conducted away from the junction than the one which it is desired to obviate.

It has furthermore been the almost invariable experience of one of the authors [1] that when an element, through exposure to combustion products or otherwise, no longer gives normal readings, the seat of the trouble lies in the five or six centimeters of the *platinum* wire immediately adjacent to the hot junction, and not in the alloy. The pure platinum sometimes seems to absorb enough volatile or other contact products, when unprotected in a furnace at very high temperatures, to alter both its resistance and its thermo-electric potential.[2] Changes of this kind are not serious when a number of control elements are constantly available, and they are usually permanently corrected by half an hour's glowing in the open air at full white heat. The glowing must be done by passing a suitable current through from end to end, and not with a Bunsen burner or gas blast.

In event of a serious accident involving an exposure of the element which can not be corrected by glowing, cutting out the exposed portion of the *platinum* wire and reconnecting will almost always restore the normal readings.

The new form of element seeks to avoid both these difficulties, it offers the advantage of an exposed junction without exposing the platinum wire, and by making the platin-rhodium cap project but little above the surface of the melting charge, it avoids excessive loss of heat by conduction away from the hot junction. In fact in this latter particular the new form enjoys a distinct advantage over the usual form of heavily protected element. It has the disadvantage of being more frail to handle, but there is little danger of anything more serious happening than the breaking of the porcelain tube, which is readily replaced.

These elements are calibrated in metal baths like the others by enclosing in a porcelain protecting tube.

The furnace, in plan, differed but little from that in use for melting-point determinations at the Reichsanstalt. In the working out, two important changes were introduced, in order to enable it to

[1] Day.

[2] Holborn & Day, Am. Journ. Sci., 4, 10, 171, 1900.

reach the higher temperatures of the mineral melting-points. A more refractory and better insulating material was substituted for fire clay in the hotter parts and the coil was wound on the inside of the oven tube instead of outside. The latter involved some little mechanical ingenuity and skill in winding, but the gain in economy and in the rapidity with which changes could be effected or constant conditions established more than repaid any additional labor in preparation.

A diagram of the furnace in section is shown in Fig. 3. It could be used for any temperature up to 1600° C. without any difficulty or especial precautions, and could be regulated to maintain a constant temperature at any particular point for long periods of time.

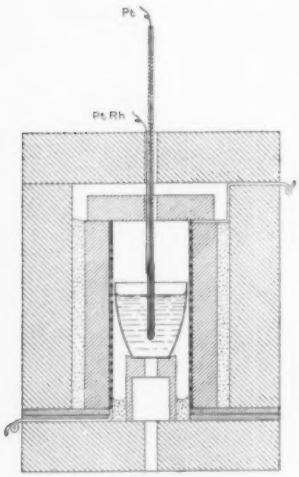

Fig. 3.

The coil, which was obtained from Dr. Heraeus, was of platin-iridium wire (90 parts Pt, 10 parts Ir), 1.5 mm. in diameter, and required about 3,000 watts to maintain a constant temperature of 1600° C. The furnace was carried at times on a 110-volt direct current street main, but accurately constant temperatures could not be depended upon without a storage battery.

The insulation in these furnaces was so perfect that shutting off or reversing the heating current at the highest temperatures did not produce a quiver in the galvanometer to which the thermo-element was connected, although the sensitiveness of the system was such that a leakage from the heating current into the thermo-element, amounting to a single microvolt (corresponding to less than 0.1°) at 1600° would have caused a displacement of about twenty divisions on the scale.

The thermo-electrical potential was measured upon a potentiometer (Wolff, Berlin, Reichsanstalt calibration) in terms of a standard cadmium cell (saturated) prepared by ourselves. Two of these cells were used interchangeably during the earlier measurements. Toward the close of the series four fresh cells were prepared for

comparison with the earlier ones and were found to agree with them within 0.0001. One of these later cells (the readings of the four were identical to the fifth significant figure) was verified by Dr. Wolff of the Bureau of Standards, by comparison with the standard Clark cells of that institution, and found to be 1.0195 V. at 20° C., assuming the legal value (United States) of the Clark cell, 1.434 V. at 15° C. Substituting the Reichsanstalt value, $Clark_{15} = 1.4328$,[1] our cells would give the normal potential difference of 1.0186 at 20°.

With the apparatus here described, the authors were enabled to command any furnace temperature up to 1600° conveniently, to regulate it quickly and with great exactness or to hold it constant for long intervals. It could also be easily arranged to introduce an oxidizing or reducing atmosphere whenever desired.

With the help of the metals mentioned, which are readily obtainable and can be used repeatedly, thermoelements or resistance pyrometers can be calibrated in any laboratory, and used for all measurements up to the limit of the Reichsanstalt scale (1150° C.)

Anorthite. First Preparation.

Date.	Element.	Electromotive Force in MV.	Temperature.	Remarks.
Oct. 7, 1903.	A	15939	1534°	Solid charge, open crucible.
" " "	A	15914	1532	" " " "
" 10, "	A	15878	1530	Covered crucible.
" " "	No. 3	16074	1533	" "
" " "	" 3	16058	1532	" "
" " "	" 3	16068	1532	" "
" " "	" 2	16095	1532	" "
			Mean, 1532°	

Second Preparation.

Jan. 16, 1904.	A	15860	1532°	Covered crucible.
" " "	A	15864	1532	
" 20 "	No. 3	15960	1533	
" " "	" 2	16102	1532	
" " "	" 2	16092	1532	
Mar. 31 "	" 3	15932	1531	1st and 2d preparations together.
			Mean, 1532°	

Melting temperature, 1532°.

[1] Jaeger u. Kahle, Wied. Ann., **65**, 926, 1898.

with no greater error than that inherent in the scale itself. Above this temperature to 1600° the continuation of the thermoelectric scale probably still furnishes the most convenient and trustworthy extrapolation which has yet been perfected.

The uniformity and certainty of this extrapolation will best be illustrated by a table taken from our measurements upon anorthite (the highest melting point we measured). The melting temperature of a mineral of very poor conductivity for heat and relatively low specific gravity is much more difficult to measure than that of a metal, but the agreement of the results tabulated above is sufficiently good to demonstrate the accuracy of the extrapolation.

These measurements were made with three different thermoelements, the individual constants of each being determined independently by separate calibration with the cadmium (321.7°), zinc (419°), silver (961.5°) and copper (1065°) melting points as described. The thermoelectric potential therefore appears to deserve entire confidence for consistent extrapolation through the 450° immediately above the present Reichsanstalt scale.

U. S. GEOLOGICAL SURVEY,
WASHINGTON, D. C., June, 1904.

THE CONDUCTIVITY OF A SPARK-GAP.

By W. P. Boynton.

SOME years ago the writer made a study of the high-frequency (Tesla) induction coil,[1] from which he concluded that the resistance of the spark-gap in the primary circuit might be of the order of from 100 ohms down to 10 or possibly even 5 ohms. Results of the same order were obtained a little earlier by Trowbridge and Richards,[2] by a method different in detail, but depending upon the same fundamental principle, namely the damping effect upon the oscillations.

Some work recently done by the writer and Mr. Ralph S. Shelley, Assistant in the Department, leads to the conclusion that the resistance of the primary spark may be, for a part of the time at least, very much less than one ohm.

This work was undertaken primarily with the purpose of verifying experimentally the theoretical conditions of resonance of such a double circuit. For such quantitative work, it was necessary to select the quantity for which resonance was to be sought. Three naturally suggest themselves; the potential difference of the terminals of the secondary condenser, the current in the secondary circuit, and the heating effect, or "mean square" current in the secondary circuit. The previous paper furnishes the basis for the mathematical discussion.

It appears the maximum attainable secondary potential is[3]

$$\frac{2C_2}{K_2} = \frac{2V_0MK_1}{\sqrt{(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2}} \tag{1}$$

and hence that the adjustment of the secondary capacity giving resonance is that which makes the expression under the radical sign a minimum, namely .

[1] Phys. Rev., VII., pp. 35-63, 1898; Phil. Mag. (5), 46, pp. 312-338, 1898.

[2] Phil. Mag. (5), 43, pp. 349-367, 1897.

[3] Loc. cit., p. 42, eq. (20); also p. 52.

$$K_2 = \frac{L_1 L_2 - 2M^2}{L_2^2} K_1, \tag{2}$$

an expression independent of the resistances of the circuits, while substituting this value gives this maximum as

$$\frac{L_2 V_0}{\sqrt{L_1 L_2 - M^2}}. \tag{3}$$

The conditions for maximum current can be easily found, but are not given, as no attempt was made to test this point.

The value for the "mean square" of the secondary current previously deduced was[1]

$$I_2^2 = \frac{n V_0^2 M^2 K_1 K_2 (R_1 K_1 + R_2 K_2)}{R_1 R_2 (L_1 K_1 - L_2 K_2)^2 + M (R_1 K_1 + R_2 K_2)^2}. \tag{4}$$

For which a maximum is found when

$$\frac{K_1}{K_2} = -\frac{R_2}{R_1} + \frac{\frac{R_2}{R_1} + \frac{L_2}{L_1}}{\sqrt{1 + \frac{R_1 M^2}{R_2 L_1^2}}}, \tag{5}$$

APPARATUS.

The primary capacity consisted of plates of window glass 12 x 16 inches coated on both sides with tinfoil, having a capacity of about .003 microfarad per plate. The secondary capacity consisted of two sheets of zinc, 4 x 6 feet, suspended vertically by white silk ribbons from a wooden frame work, forming an air condenser of adjustable capacity. The ratio of the capacities was measured directly by the bridge method, using intermittent current (110 volts) and telephone.

The primary consisted of five turns of heavily insulated No. 4 wire wound in an open spiral about 9 cm. long and of the same diameter; the secondary consisted of 104 turns No. 20 wire wound on a glass cylinder, forming a solenoid 13 cm. long and 4.3 cm. in diameter. Their inductances were determined by comparison with a standard of self-inductance, whose computed value, .00419 henrys, had been checked by comparison with measured capacities. The

[1] Loc. cit., p. 47, eq. (26).

Anderson-Maxwell method as modified by Fleming [1] was employed for the self-inductances, using a condenser as an intermediary, and the mutual inductance was determined from the self-inductance of the secondary coil by Maxwell's method,[2] adopting Fleming's modification suggested in the paper last referred to. The values were also computed as a check. They were found to be

$$L_1 = 2{,}000 \text{ cm.},$$

$$L_2 = 130{,}000 \text{ cm.},$$

$$M = 5{,}400 \text{ cm.}$$

The spark gaps employed were zinc knobs of about 2 cm. diameter mounted on glass standards.

Experimental.

A spark gap was placed in parallel with the secondary condenser, and readings of the spark length taken for a series of different distances between the plates of the condenser. With the arrangement employed, namely 8 condenser plates in the primary condenser, and a primary spark of about .13 cm., a distinct maximum of about 1.2 cm. was observed when the plates were about 8 cm. apart. A direct comparison showed the secondary capacity to be 1/71.5 of the primary. The ratio computed by substituting the values of the inductances in formula (2) was 1/83. This discrepancy could easily have been made to disappear by moving the plates slightly farther apart, which would have produced no observable change in the spark length. Substituting the values of the inductances in formula (3) shows that the secondary potential cannot be expected to exceed 8.56 times the primary. The ratio of the spark lengths, $1.2/.13 = 9.22$, is entirely consistent with this conclusion as the potentials are very nearly represented by a linear function of the spark length, so that

$$\frac{V_2}{V_0} = \frac{S_2 - a}{S_1 - a}, \tag{6}$$

in which S_1 and S_0 represent the spark lengths and the not impos-

[1] Phil. Mag. (6), 7, p. 536. 1904.

[2] Elec. and Mag., Vol. II., pp. 397-398 (3d. edition).

sible value $a = .0115$ would give exactly the computed ratio, 8.56.

This preliminary test serves to indicate that the apparatus functions normally.

Measurements of the "mean square" current were made by a hot wire ammeter similar to that described in the previous paper.[1] Readings of this instrument were taken as in the case of the potential measurements for a series of distances between the plates of the secondary condenser, then these were set at the distance seeming to correspond to the maximum current, and the ratio of primary and secondary capacities determined. This was done for different values of the primary capacity, as, shown in the following tabulation:

No. plates in primary	2	3	4	5	6	8
K_1/K_2	22	29	31	34	35.5	38

For comparison the values of K_1/K_2 are plotted from eq. (5) in the accompanying figure as a function of R_2/R_1. A comparison of the

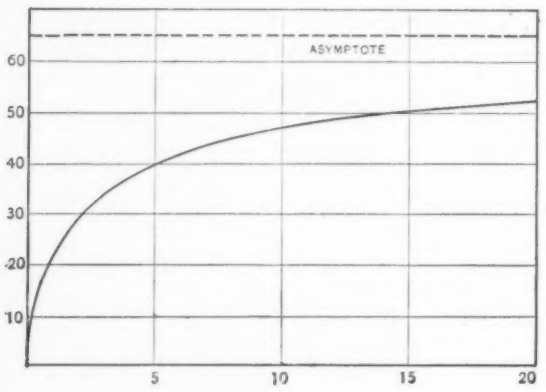

Fig. 1.

observed ratios with the tabulation indicates that the values of R_2/R_1 lie between 1 and 5, or since the measured value of R_2 is 2.07 ohms, the value of R_1 would seem to lie between 2.07 and .415 ohms. As the latter value is not very different from the ohmic resistance of the metallic circuit, the implication is that the spark

[1] Loc. cit., p. 53.

gap may offer a resistance of much less than an ohm to the passage of a large oscillatory discharge.

Discussion.

The mathematical theory as previously developed has assumed that all resistances are metallic, that is, constant. On this assumption an oscillatory system ought to give a wave train with a considerable number of observable oscillations. Such a system of metallic conductors having a single period of oscillation was investigated by Webster,[1] who records measurements of the period of perhaps 30 oscillations of the system.

But a conducting gas, such as a spark gap, does not have a constant resistance, such an assumption being only a first approximation. A second approximation which might prove useful would be the assumption that the resistance of such a spark has for a short time a definite, finite steady resistance, suddenly changing at the end of that short time to an infinite resistance. Apparently such an assumption would explain the various phenomena, the resonance effect being determined by the instantaneous small value of the resistance, the effect recorded in the previous paper being due to a time-average effect, while the number of spark-records obtained in various photographs would be related both to the instantaneous value of the resistance, and to the time during which this value persisted.

The data taken at this time are not sufficient for a satisfactory quantitative discusssion of this new assumption, hence an attempt will be made to apply it to older data.

In the indefinite integral of the square of an oscillatory function of the form

$$e^{-\alpha t}(A \cos \beta t + B \sin \beta t)$$

the principal term is[2]

$$-\frac{(A^2 + B^2)e^{-2\alpha t}}{4\alpha}.$$

If the integration be taken not from 0 to ∞, but from 0 to a finite limit τ, the definite integral will be

[1] Phys. Rev., VI., pp. 297–314, 1898.

[2] Loc. cit., p. 44.

$$(7) \qquad (1 - e^{-2a\tau})\frac{A^2 + B^2}{4a},$$

which becomes, if $2a\tau$ is small

$$(8) \qquad 2a\tau\frac{A^2 + B^2}{4a} = (A^2 + B^2)\tau/2.$$

Applying this result to the secondary potential, and to the secondary and primary currents,

$$(9) \qquad \int_0^\tau V_2^2 dt = \frac{(A_2^2 + B_2^2)\tau}{2K_2^2} + \frac{(C_2^2 + D_2^2)\tau}{2K_2^2},$$

which becomes, neglecting B_2^2 and D_2^2 as small, and noting that $C_2^2 = A_2^2$

$$(10) \qquad \int_0^\tau V_2^2 dt = A_2^2\tau/K_2^2.$$

$$(11) \qquad \int_0^\tau I^2 dt = (\alpha^2 + \beta^2)(A^2 + B^2)\tau/2 + (\gamma^2 + \delta^2)(C^2 + D^2)\tau/2,$$

which becomes for the secondary current, neglecting α^2, γ^2, B_2^2 and D_2^2 as small and remembering that $C_2^2 = A_2^2$

$$(12) \qquad \int_0^\tau I_2^2 dt = (\beta^2 + \delta^2)A_2^2\tau/2,$$

and for the primary current, where C_1 preponderates over the other coefficients.

$$(13) \qquad \int_0^\tau I_1^2 dt = \delta^2 C_1^2\tau/2.$$

If, for the sake of definiteness, we assume that τ is the time required for m half-oscillations of the primary current, then evidently

$$(14) \qquad \delta\tau = \pi m, \qquad \tau = \pi m,$$

from which

$$\int_0^\tau V_2^2 dt = \pi m A_2^2/\delta K_2^2,$$

$$(15) \qquad \int_0^\tau I_2^2 dt = \pi m(\beta^2 + \delta^2)A_2^2/2\delta,$$

$$\int_0^\tau I_1^2 dt = \pi m\delta C_1^2/2.$$

Substituting the values of β, δ, C_1, and A_2, and introducing n, the number of discharges per second of the primary condenser, the following average values are obtained:

$$\bar{V}_2^2 = \frac{\pi n m M^2 K_1^2 V_0^2 \sqrt{L_1K_1 + L_2K_2 + \surd(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2}}{\surd 2[(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2]},$$

$$(16)\quad \bar{I}_2^2 = \frac{\pi n m M^2 K_1 K_2 V_0^2 (L_1K_1 + L_2K_2) \sqrt{L_1K_1 + L_2K_2 + \surd(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2}}{2\surd 2(L_1L_2 - M^2)[(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2]},$$

$$\bar{I}_1^2 = \frac{\pi n m K_1 V_0^2 \sqrt{L_1K_1 + L_2K_2 - \surd(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2}\,[L_1K_1 - L_2K_2 + \surd(L_1K_1 - L_2K_2)^2 + 4M^2K_1K]}{16K_2(L_1L_2 - M^2)[(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2]}.$$

In these approximate equations the "effective" readings appear to be conditioned not upon the resistance of the spark-gap, but upon m, the number of half-oscillations it permits to pass. The first two quantities also are very simply related, having the ratio

$$(17)\quad \frac{\bar{V}_2^2}{\bar{I}_2^2} = \frac{2}{(\beta^2 + \delta^2)K_2^2} = \frac{2K_1(L_1L_2 - M^2)}{K_2(L_1K_1 + L_2K_2)}.$$

Substituting numerical values in this equation we have, reducing to practical units and extracting the square root,

$$(18)\quad \frac{\bar{V}_2}{\bar{I}_2} = 5{,}300.$$

Fig. 2 shows data of the discharge with a 2 mm. spark,[1] plotted as functions of the excitation, $\bar{I}_2$ being in hundredths of an ampere, $\bar{V}_2$ in hundreds of volts, while the ratio $\bar{V}_2/\bar{I}_2$, computed from the curves, not the points, shown by the broken line, appears very nearly constant, ranging for the most part between 5,500 and 5,000.

Equations (16) may also be used for the computation of m. Substituting the values of the known constants, and reducing to practical units, they give

[1] Loc. cit., pp. 57–58; Table III., Series II. and III.

$$m = \overline{3,340} V_2^2/V_0^2,$$

(19) $$m = 9.42 \times 10^{10} \overline{I_2}^2/V_0^2,$$

$$m = 5.18 \times 10^8 \overline{I_1}^2/V_0^2.$$

Fig. 3 gives values for m from the data just referred to. This indicates a number of half-oscillations varying from 2 to 8, as shown by data from the secondary circuit, or including one point not in the figure, up 16 as shown by data from the primary circuit. As an interesting coincidence, upon reference to the original spark

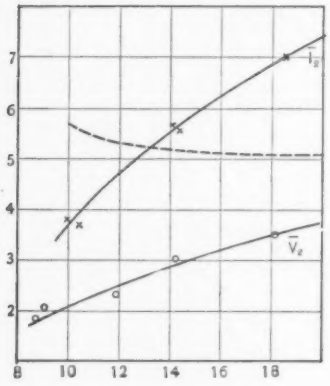

Fig. 2.

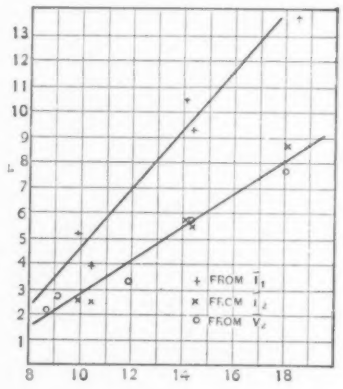

Fig. 3.

photographs, one can count in many of them 8 striations, rarely 9, but often less. These however were taken under such different conditions that their evidence on this evidence on this point is not conclusive.

Conclusion.

Trowbridge and Richards[1] specifically speak of the "spark from a large condenser" as the condition of low resistance of spark gap. The justice of this remark is apparent from the fact that in the earlier experiments with a spark from a condenser of .0016 M.F. capacity only the data taken with the 2 mm. spark were available for the purposes of this discussion, the others showing irregular and abnormally large values of the ratio $\overline{V}_2/\overline{I}_2$, while in the present investigation capacities from 4 to 15 times as large were used,

[1] Loc. cit.

namely, from .005 to .023 M.F., when the low resistance effect became strikingly manifest.

The two sets of equations deduced appear to be adapted to the treatment of the two types of experimental data, the old equations perhaps better expressing the conditions when the condenser is small or the spark long and thin; the newer ones better adapted to the case where the primary condenser is large, and the spark relatively short.

An attempt to find a maximum value of $\overline{I_2^2}$ in equation (16) by differentiation gave forms too complicated for solution; but the method of trial indicates a maximum in the neighborhood of $K_1/K_2 = 90$ for the later apparatus. This by no means accords with the experimental results, which run from 22 to 38; but as the resonance is conditioned upon instantaneous rather than average values, it is not surprising that the newer formulæ, which take account primarily of the time of continuance, should fail to agree with experiment.

The complete mathematical theory would of course treat the resistance of the spark gap as a variable, a function possibly of the time, and thus indirectly a function of the effective current, or the energy of the field.

PHYSICAL LABORATORY, UNIVERSITY OF OREGON,
June, 1904.

EXPERIMENTS ON RESONANCE IN WIRELESS TELEGRAPH CIRCUITS.

BY GEORGE W. PIERCE.

I. INTRODUCTION.

TO be enabled to foresee the extent to which confusion may be avoided in the simultaneous operation of a great number of wireless telegraph circuits, it is important to ascertain quantitatively the relative energy communicated between various dissonant circuits compared with the energy communicated between circuits in unison; that is, to ascertain the shape of the curve made by plotting the received energy (or the current in the receiving circuit) as a function of the periods of the two circuits.

Two forms of sending circuit capable of resonance are shown in the accompanying figures. The first of these forms — the "electromagnetically coupled type," Fig. 1 — is seen to consist essentially of a Tesla coil with one terminal of the secondary connected to earth, while the other terminal is attached to a more or less nearly vertical wire carried by a mast.

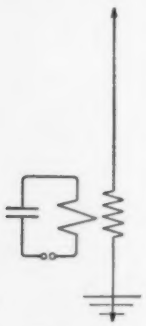

Fig. 1.

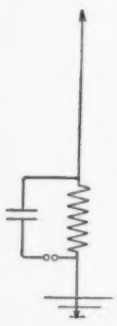

Fig. 2.

The second form — the "directly coupled type" — is shown in Fig. 2. It is seen to contain an autotransformer instead of the Tesla arrangement of Fig. 1.

The corresponding receiving circuits employed by different inventors resemble more or less closely the sending circuit with, however, a coherer and its appurtenances or other form of detector substituted for the spark gap.

The important advantage for resonance of the two types here represented consists in the direct grounding of the antenna, and in

the production of the oscillation in the antenna by the inductive action of a condenser discharge in an auxiliary circuit of small damping.

The two forms of *sending* circuit shown in Figs. 1 and 2 seem to have been first clearly described by Professor Ferdinand Braun[1] of Strassburg, and both forms are covered by German Patent No. 111,578, issued Oct. 14, 1898, to the "Gesellschaft für drahtlose Telegraphie," basing its claim on Braun's discovery. The priority of discovery of these two types of sending circuit is attributed to Braun by M. Wien,[2] who says, "Die Bedeutung dieses Fortschrittes wird vielleicht am besten dadurch characterisiert, das so wohl Marconi als Slaby inzwischen zu der Braun'schen Methode der gekoppelten Systeme uebergegangen sind."

While Braun was active in developing the two forms of inductive sending circuits represented in Figs. 1 and 2, Marconi utilized the same principle in the construction of a syntonic *receiving* station. In 1898 he saw the importance of the direct grounding of the receiving antenna, and his English Patent No. 12,326 of June 1, 1898, provides for the location of the coherer in a closed condenser circuit inductively connected with the antenna. His receiving circuit is essentially of the form of Fig. 1 with the coherer in place of the spark gap, and his patent is of slightly earlier date than the corresponding patent of Braun covering the sending circuit.

Later, on the 21st of March and the 26th of April, 1900, Marconi has been granted English patents on a sending circuit closely resembling that of Fig. 1, with, however, the addition of a variable inductance in the antenna for the easy adjustment of the mast circuit to resonance with the condenser primary.

The theory of resonance in wireless telegraphic circuits has been treated by a number of writers. The simple form of circuit, used by Marconi prior to 1898, consisting of vertical wire, spark-gap or coherer, and ground has been discussed theoretically by V. Bjerknes[3] and M. Abraham.[4] The theory of the sending and receiving

[1] Phys. Zeit., 2, 372, 1901; and 3, 143, 1901–2.
[2] Drude's Ann., 8, 686, 1902.
[3] W. A., 55, p. 120, 1895.
[4] Phys. Zeit., 2, p. 329, 1901.

circuits of the type shown in Fig. 1 has been presented by M. Wien,[1] while the theory of the type shown in Fig. 2 has been made the subject of a recent paper by M. Abraham.[2]

Also a number of experiments have been published that treat of certain parts of the problem of resonance in such circuits. P. Drude[3] has made extensive experiments on the period of electrical vibrations of spools of wire with antennæ or metal plates attached. F. Braun,[4] by hanging Geissler tubes in various positions along the antenna of a single circuit has obtained evidence of nodes and loops when the side circuits are properly adjusted. C. A. Chant[5] has made similar investigations with a quantitative instrument in place of the Geissler tubes. James E. Ives,[6] by means of a hot-wire ammeter in the lead to earth has obtained some curves showing resonance of the antenna with the condenser circuit connected to it as in Braun's directly connected system (Fig. 2).

The researches made by the above investigators have all been experiments at the sending station, where the current in the antenna is large. In my own experiments this current, as measured by a hot-wire ammeter, sometimes attains an average value of two or three amperes.

Recently A. H. Taylor[7] has published some results of direct measurement of the current at the *receiving* station. By the use of a bolometer as a receiving instrument, Mr. Taylor has adjusted a pair of circuits to resonance, and with a fixed sending circuit giving out waves of approximately constant intensity and constant period has plotted the current in the receiving circuit when the inductance in the receiving antenna was varied. Mr. Taylor's article contains also interesting lecture-room demonstrations of the phenomena.

In the experiments described in the present paper the relative energy in the receiving circuit has been measured directly for several variations of the capacity and inductance of both the sending and

[1] W. A., 61, p. 151, 1897; and D. A., 8, p. 686, 1902.
[2] Phys. Zeit., 5, p. 179, 1904.
[3] D. A., 9, p. 293, 1902; and 11, p. 957, 1903.
[4] Phys. Zeit., 3, p. 143, 1901–2.
[5] Am. Jour. Sci., 1904.
[6] El. World and Eng., Feb. 6, 1904.
[7] PHYS. REVIEW, 18, p. 239, April, 1904.

the receiving circuit. The measurements show that the phenomena may be made to give sufficient regularity to permit of quantitative study.

II. Apparatus and Method.

Form of Circuit. — The present experiments are confined to circuits of the electromagnetically connected type of Braun and Marconi. The sending circuit is shown at *S* in Fig. 3.

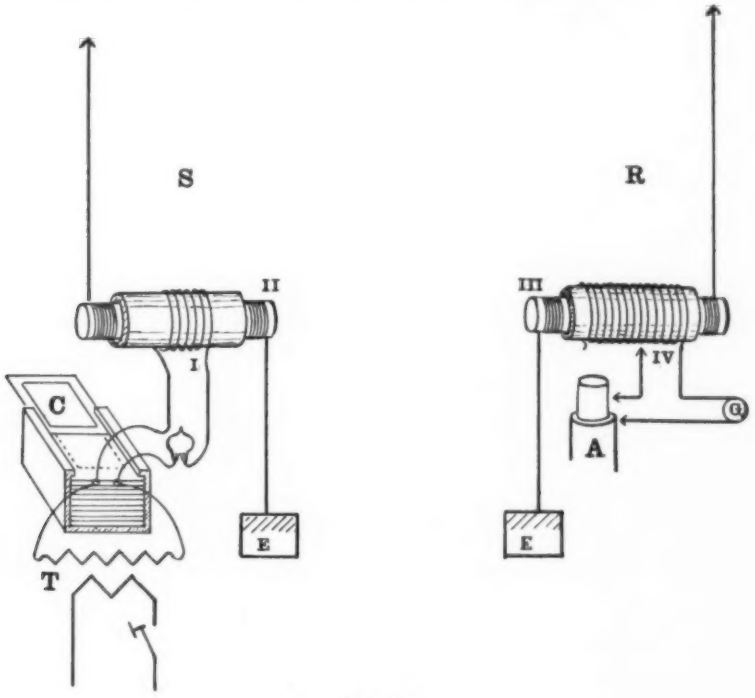

Fig. 3.

C is a variable glass plate condenser, charged by a step-up transformer *T*, operating on the 110-volt alternating light circuit. The transformer is capable of giving a potential of 10,000 volts, but by means of a rheostat in the primary this voltage was usually reduced to about 5,000. The condenser discharges through a Cooper-Hewitt mercury interrupter and through a primary of a few turns of coarse wire (diameter = 2 mm.) wound on a tube of hard rubber 1.25 cm. thick and 13 cm. external diameter (Circuit I.). Into the bobbin on which the primary is wound various secondaries can be

inserted. The secondaries were of insulated wire wound on thin rubber tubes (2 mm. thick) and 10.4 cm. external diameter. Two such secondaries were used with the following dimensions:

No. of Turns.	Diam. of Wire, cm.	Length of Solenoid, cm.	Inductance Henries.
240	.104	46	125×10^{-5}
115	.157	41	31×10^{-5}

The secondary at the sending station, together with its antenna and ground connection, is referred to in the descriptions as Circuit II. At the receiving station R the inner coil with one end grounded and the other connected to the antenna (Circuit III.) had the same number of turns and the same dimensions as the corresponding coil at the sending station. The closed circuit electromagnetically connected with it (Circuit IV.) consisted of a variable air condenser in series with the receiving instrument and a bobbin on which the number of turns could be varied by means of a sliding contact.

The sending and receiving circuits were set up at separate small houses 25 meters apart. There were no wires or other metallic connections between the stations. The grounds consisted of pieces of wire netting (1 meter by 3 meters) buried in a horizontal position 40 cm. below the surface of the earth. The antennæ were copper wires 2 mm. in diameter passing in porcelain tubes through the walls of the buildings. These antennæ were usually about 7 meters high.

The Condensers. — The variable capacity at the receiving station consisted of four cylindrical air condensers each made of two concentric drawn tubes of brass 2 mm. thick. These tubes were made by the American Tube Works and are very approximately of uniform bore. They were straightened and freed from the small distortions by turned brass castings slipped around outside the outer and inside the inner tube of each condenser. These tubes are 60.8 cm. long. The outside diameter of the inner tube is 14.62 cm. The distance between the two tubes was obtained by setting a condenser on a block of paraffin and finding the weight of water required to fill the interspace. For the condenser A this distance was found to be .147 cm. From these data the capacity of A was computed to be

1.69×10^{-3} microfarads.

The other three air condensers did not differ materially from *A*.

The inner tube was supported on plugs of hard rubber and could be slid in and out of the outer tube without much motion sidewise — certainly without a displacement of the axis by as much as fifteen per cent. of the distance between the cylinders, and hence with a change of less than one per cent. in capacity.

The connections to the inner cylinder was made by means of a stiff copper brush. The capacity at the receiving station was varied by throwing in and out these air condensers as wholes or by varying any one of them by withdrawing the inner cylinder and reading on a paper scale the number of centimeters of length left overlapping — 1 cm. being equal to 2.77×10^{-5} microfarads.

The capacity *C* at the sending station consisted of two condensers made with plates of glass as dielectric. The number of such plates could be varied, and final small variations could be made by sliding back the top plate of one of them. The capacity of one of these glass condensers was measured for approximately the frequency at which it was to be used by comparing it with the known air condensers. The comparison was made by a resonance method as follows: Two parallel rectangular loops of wire 20 meters apart were hung up in the laboratory. One of these loops containing a condenser and spark-gap, or better a Hewitt mercury interrupter, was used to produce the waves of the proper period. The other loop, used as a receiving circuit, was adjusted to resonance with the sending circuit, first by the use of the variable air condenser, then by the glass condenser, plus a part of the air condenser. The air condenser being known, the capacity of any number of plates of the glass condenser could be determined. The other of the glass condensers used at the sending station was subsequently measured by comparisons obtained in the experiments that are described later in this account.

The Receiving Instrument. — Fig. 4 shows the instrument used in measuring the energy in the condenser circuit (Circuit IV.) at the receiving station. It is a slightly modified form of the oscillating current galvanometer devised by Fleming. A thick-walled hard rubber tube with an internal bore 1 cm. in diameter is mounted on a metallic base provided with leveling screws. In this tube is hung

the suspension, which consists simply of a plane glass galvanometer mirror, 3 mm. in diameter, backed up by a thin disc of silver of approximately the same diameter. The mirror and disc are hung by a fine quartz fiber. Near the disc *M* is a small coil *C* making an angle of about 45° with the plane of the mirror at rest. This coil (diameter 9 mm.) consists of 30 turns of insulated wire (.11 mm.

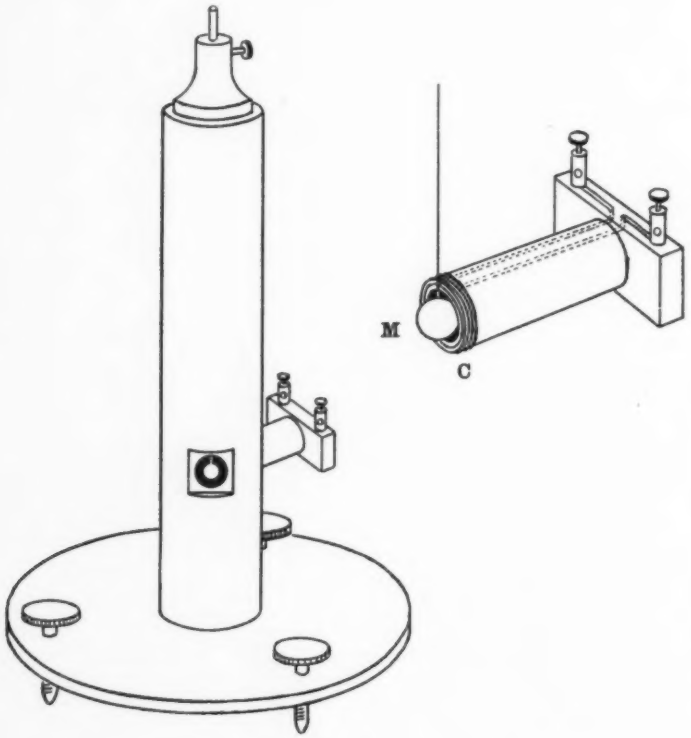

Fig. 4.

in diameter) and is mounted on the end of a thin hard rubber tube capable of sliding back and forth in the supporting case so that the distance between the mirror and coil can be varied, thus giving the instrument a very wide range of sensitiveness. The coil is connected by binding posts directly in series with the condenser circuit at the receiving station. Oscillations in this circuit conducted through the coil induced oscillations in the suspended disc, which is thus repelled, and tends to set itself at right angles to the plane of the coil.

The period of swing one way is 3.4 seconds; and the mirror comes to rest after four or five oscillations. The resistance of the coil is 1.33 ohms. Its self inductance measured by Rayleigh's bridge with 1,200 alternations per second is 1.05×10^{-5} henries. This instrument is satisfactorily sensitive for the experiment. In fact, to keep the deflections on the scale when the circuits were in the neighborhood of resonance it was necessary to have the plane of the coil of the instrument at a distance of over 1 cm. from the center of the suspended disc. In order that the distance between the coil and the disc might not be changed by accidental jarring the instrument was screwed rigidly to a heavy shelf attached to a brick wall.

To avoid electrostatic disturbances of the instrument a small quantity of radioactive substance (radium bromide) was put inside the case to dissipate the charge acquired by the mirror in the process of hanging the suspension. This proved a great convenience, and after its use the instrument remained quiet within .1 or .2 mm., and kept the same zero for weeks.

For deflections so small that the change in the mutual inductance between the coil and the disc is negligible simple theoretical considerations show that the deflection is proportional to the square of the current in the coil. It is not presumed that such an instrument could be used as a receiver for wireless signals at any great distance, but without any very great precaution in its use it proves to be applicable to the rapid and consistent measurement of feeble currents of high frequency.

The Hewitt Mercury Interrupter. — In a previous paper[1] I have pointed out the advantages of the mercury interrupter both for uniformity and intensity in comparison with the spark-gap when used to interrupt a condenser discharge. The action of the mercury interrupter is, however, influenced by the temperature outside the bulb. The amount of this influence is shown in Section IV. of this paper.

In the present experiments, where the interrupter was operated in an unheated wooden building in which the temperature ranged from — 20° to 15° C., it was necessary, on account of the effect of temperature, to use the mercury interrupter in a thermostat, which

[1] Pierce, Proc. Am. Acad., Vol. XXXIX., No. 18, p. 389, Feb., 1904.

consisted of an electrically heated bath of oil with the temperature automatically regulated. The following sets of readings (Table I.) taken at various times for different sensitivenesses of the instrument give an idea of the uniformity that may be attained in such observations. The readings are in centimeters with a scale-distance of 90 cm. Each column gives successive readings under the same conditions.

TABLE I.

To Show Uniformity.

10.8	37.5	28.7	11.8	37.7	11.4
10.9	37.2	28.0	11.2	37.2	11.4
11.4	37.0	28.1	11.8	39.0	11.2
11.4	36.1	28.3	11.9	36.7	11.2
11.2	38.9	28.5	11.7	36.2	11.1
11.1	38.0	27.8	11.6	36.8	10.8
10.8	38.3	29.4	11.4	37.8	11.0
11.0	38.1	28.5	11.2	39.8	11.0
11.0	38.1	29.5	11.4	38.8	11.2
11.2	38.1	29.0	11.4	37.8	11.0

These deflections for various mercury interrupters were from five to seven times as large as could be obtained by the spark-gap operating at the same potential. For low discharge potentials (spark 1 to 2 mm.) by increasing the sensitiveness of the receiving instrument it was possible also to get uniform readings with the spark-gap, but for a long series of observations the mercury interrupter is much more satisfactory. This advantage is, however, partially offset by the annoyance caused by the breaking of the bulb when exposed too long to a heavy current—a difficulty that may possibly be overcome by correct annealing of the glass.

III. RESONANCE CURVES.

Experiment I. Resonance Curves with the Secondaries with 240 Turns.—The two mast circuits were made similar, each containing one of the secondary coils of 240 turns, described on page 200. On the outside bobbins, Fig. 3, there were 9 turns at the sending station and 20 turns at the receiving station. The mast height was 7 meters of bare wire 2 mm. in diameter. The family of curves

plotted in Fig. 5 were obtained by varying the capacities at the sending and receiving stations. Each curve corresponds to a fixed sending capacity; the individual points on any one curve correspond to various capacities of the receiving condenser.

The process of taking readings was as follows: The glass condenser at the sending station was set at a given small value (2 plates), the air-condenser *A* at the receiving station was set at 6 cm. (length of cylinder overlapping), the primary circuit of the transformer was closed by a switch operated by a string. This switch was left closed during the outward swing of the mirror, and the throw was read. The value of the throw is plotted as the first

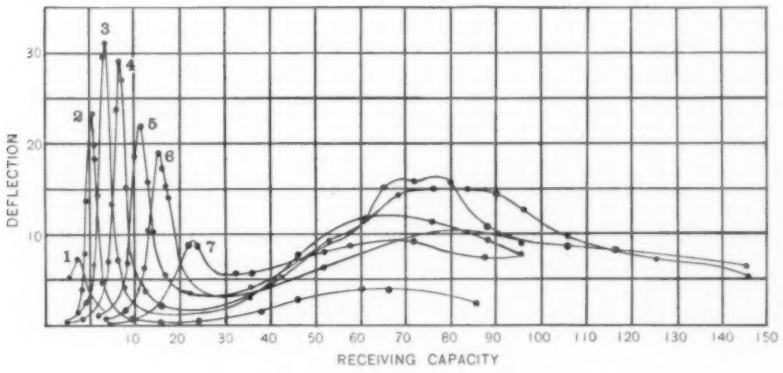

Fig. 5.

point on Curve 1. The sending circuit remaining constant, successive deflections were taken with the receiving condenser set at 8, 10 and 12 cm. respectively. These four values determine Curve 1, in which deflections are plotted against receiving capacity. From Curve 1 it is apparent that the random choice of the sending capacity is not correct for resonance with the mast circuits.

For Curve 2, the condenser at the sending station was set at 3 plates. Keeping this constant, deflections were taken for various values of the receiving capacity ranging from 6 to 96 cm. of the cylindrical condensers. Curve 2 thus obtained shows a larger maximum than Curve 1; which indicates that the second adjustment of the sending circuit gave a nearer approach to resonance between the sending condenser circuit and the mast circuits. Sub-

sequent increases of the sending capacity by small stages gave the Curves 3, 4, 5, 6 and 7. Of these Curve 3 is the best. It was obtained with a sending capacity of 3.3 plates of the glass condenser. (The various capacities here designated are given in microfarads in Table II.)

An examination of Fig. 5 shows, however, that the deflections,

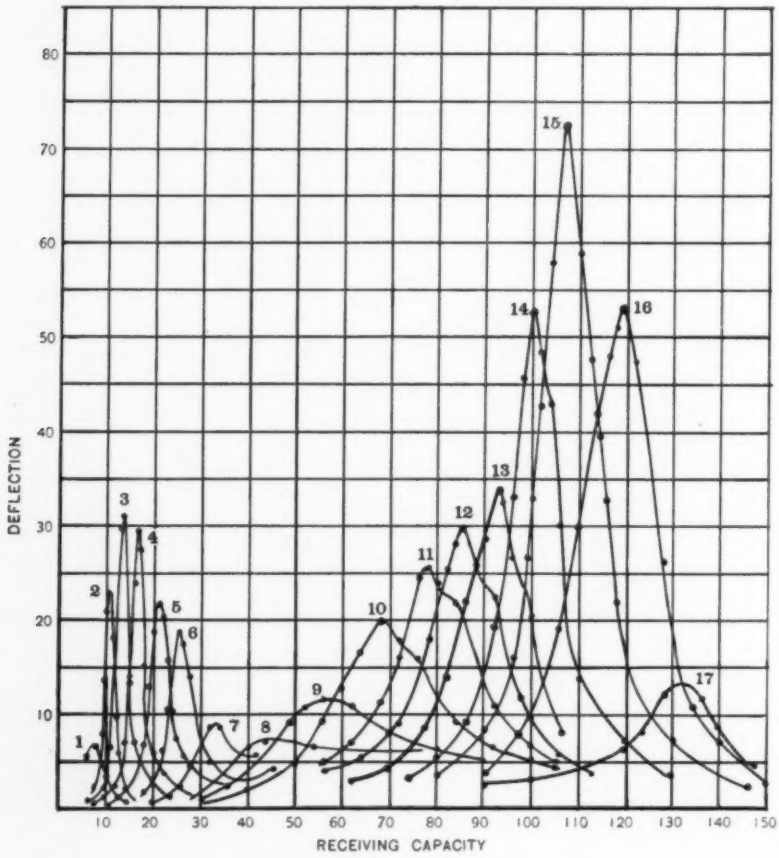

Fig. 6.

after falling for receiving capacities between 30 and 40, rise again in the region between 70 and 100. This was taken to indicate that the sending condenser circuit in the case that gave Curve 3 is in resonance with a *harmonic* of the mast and not with its fundamental period.

To test this view, the sending capacity was increased further by small stages, and the Curves 8 to 16, Fig. 6, were taken. In this figure Curves 1 to 7 are also plotted, as in Fig. 5, except that their second maxima are omitted in order to prevent confusion with the subsequent curves.

Curve 15 of Fig. 6 examined for percentage decrease of deflection with percentage dissonance, is seen to give resonance more strongly marked than Curve 3.

The 17 curves of Fig. 6 were all taken in a single series of observations in a single day. Their relative values were then checked up by taking the average of three observations at each of the maxima, so as to be sure that no progressive changes occurred in the action of the interrupter during the observations.

One might ask whether Curve 15 is correct for resonance or whether we have again merely resonance of the sending capacity circuit with a harmonic of the mast circuit. The answer is obtained by noticing that Curves 3 and 15 have their maxima at 13 and 107 cm. of the receiving condensers. These numbers are approximately

TABLE II.

Curve No.	Sending Capacity in Microfarads.	Optimal Rec. Capacity in Microfarads.	Ratio, *S.R.*
1	12.3×10^{-4}	2.21×10^{-4}	5.6
2	17.5	2.90	6.0
3	20.5	3.75	5.5
4	25.6	4.70	5.5
5	30.7	5.80	5.3
6	37.0	6.90	5.4
7	54.0	9.12	5.9
8	73.5	12.2	5.9
9	103.5	15.5	6.6
10	111	18.8	5.9
11	124	21.3	5.8
12	130	23.5	5.5
13	137	25.7	5.3
14	158	27.7	5.7
15	164	29.7	5.5
16	178	33.0	5.4
17	200	36.5	5.5
			Mean 5.7

in the ratio of 1 : 9 (*e. g.*, 12 : 108). The *periods* in these two cases are, therefore, in the ratio of 1 : 3, which is the ratio of the period of the first odd harmonic to that of the fundamental. The same ratio could not obtain for any other two odd harmonics, hence Curve 15 is the case of resonance with the fundamental period of the mast circuits.

In the descriptions given above of the curves of Fig. 6 the capacities are stated in arbitrary units. Table II. contains these values in microfarads. The second and third columns give the sending capacities and the corresponding optimal receiving capacities for the several curves of Fig. 6. The fourth column contains the ratio of these two capacities.

The inductances were kept constant throughout the series and had the following values in henries :

	Henries.	Method of Measuring.
L_2 and L_3	125×10^{-5}	— Maxwell's Bridge.
L_4	6.0×10^{-5}	Resonance, cf. p. 210.
	5.95×10^{-5}	Rayleigh's bridge.
L_1	1.05×10^{-5}	Assuming it to be $L_{4/5.7}$.
M_{12}	4.85×10^{-5}	Comparison with Earth Inductor.
M_{34}	10.9×10^{-5}	" " " "

The approximate constancy of the ratios in the fourth column of Table II. shows that for the particular circuits employed in Experiment I. the several curves of Fig. 6 are the curves of resonance between the two condenser circuits I. and IV. This means that the predominating oscillation in the mast circuits II. and III. is a forced vibration with approximately the period determined by the closed circuit I. The characteristic oscillation of the mast circuits themselves, on account of high damping, does not persist strongly enough to have much effect in determining the *period* of the train of waves that carry the energy. The *intensity*, however, is considerably influenced by the resonant action of the masts. The great intensity obtained in Curve 15 is the effect of tuning the sending condenser circuit to resonance with the masts. For the maximum point on this curve the receiving condenser circuit is also in resonance with the masts. The *purity* of the wave is also enhanced

by unison of the discharge circuit with the masts. For the sending circuit in the condition to give Curve 15 there is no evidence of more than one maximum, even when examined over a range of several octaves. Thus with equal mast circuits harmonics appear ouly when the condenser circuit I. is not in resonance with the mast circuits II. and III.

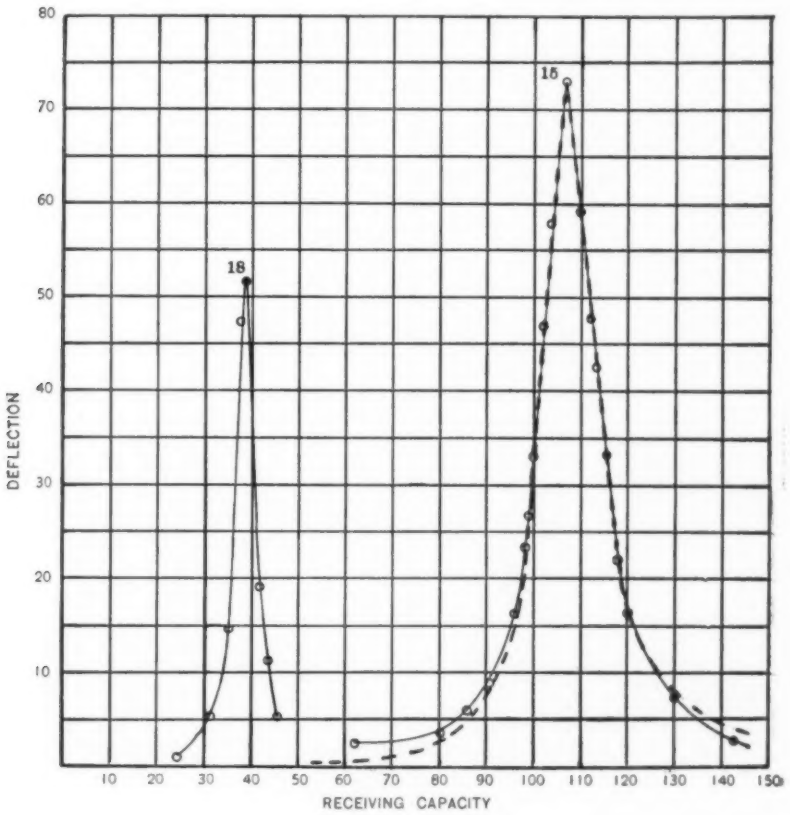

Fig. 7.

Experiment II. Determination of the Fundamental Wave Length for Curve 15.— To determine the wave-length of the apparatus in the condition of best resonance, a known inductance was put in the circuit with the air condenser and instrument at the receiving station, and readings were taken for several successive values of the receiving condenser. The results are plotted as Curve 18, Fig. 7, with a

maximum at $C = 38.5 = 38.5 \times 2.77 \times 10^{-11}$ microfarads. This curve compared with 15, which is also plotted in Fig. 7 gives the data for calculating the fundamental wave-length. The oscillations producing curves 17 and 14 have the same period, since the sending circuit is the same in the two cases; therefore,

$$T = 2\pi\sqrt{L \times 107 \times 2.77 \times 10^{-11}}$$
$$= 2\pi\sqrt{(L + L') \times 38.5 \times 2.77 + 10^{-11}}. \quad (1)$$

In this equation L is the unknown inductance of the leads and instrument, L' the known inductance of the added coil. The coil L' was accurately wound, and its inductance was computed from its geometrical dimensions by the aid of Maxwell's formula. This coil was also measured in comparison with a calculated air condenser, by photographing with a revolving mirror the period of the discharge of the condenser through the coil. These two methods agreeing within one per cent., gave

$$L' = .000106 \text{ henries.}$$

This value substituted in equation (1) gives for the inductance of the leads

$$L = \frac{38.5 \times .000106}{107 - 38.5},$$
$$= 6.00 \times 10^{-5} \text{ henries.}$$

A subsequent measurement of this quantity on Rayleigh's bridge with 1,200 alternations per second gave

$$L = 5.95 \times 10^{-5}.$$

The agreement is satisfactory. The substitution of this value of L in equation (1) gives for the period

$$T = 2.66 \times 10^{-6} \text{ seconds.}$$

Whence the wave-length

$$\lambda = vT = 798 \text{ meters.}$$

Discussion of the Resonance Obtained in Curve 15.— In the results plotted as Curve 14 it is seen that the deflection (*i. e.*, the square of

the current) falls to one half its maximum value when the receiving capacity departs by 7 per cent. from the capacity of resonance. Simple theoretical discussion of the case may serve to discover the conditions that have prevented sharper resonance and may serve to suggest methods for improving these conditions. As a rough approximation let us suppose that the condenser circuit containing the instrument at the receiving station has a resistance R, a self inductance L and a localized capacity C, and that there is impressed upon it a sinusoidal electromotive force $E \sin \omega t$, and let us take no account of the neighboring mast circuit, then the square root of the mean square current through the instrument is

$$I = \frac{E/\sqrt{2}}{\sqrt{R^2 + \left(\frac{1}{c\omega} - L\omega\right)^2}}. \tag{1}$$

Second, let us suppose that the deflections of the receiving instrument are proportional to I^2;

$$D = \frac{kE^2}{R^2 + \left\{\frac{1}{c\omega} - L\omega\right\}^2}. \tag{2}$$

In Experiment I., L was kept constant, and by Experiment II. it was found to have the value

$$L = 6.00 \times 10^{-5} \text{ henries.}$$

Also in obtaining Curve 14 the incident waves were kept constant in intensity and frequency, so that E is a constant, and

$$\omega = \frac{1}{\sqrt{LC_m}},$$

$$L\omega = \frac{\sqrt{L}}{\sqrt{C_m}} = \sqrt{\frac{6.0 \times 10^{-5}}{29.7 \times 10^{-10}}} = 144.$$

Then from (2)

$$D = \frac{kE^2}{R^2 + 144^2 \left\{\frac{C_m}{C} - 1\right\}^2}. \tag{3}$$

Equation (3) contains two unknown constants kE^2 and R^2. A trial shows that with proper values of these constants equation (3) gives a fair approximation of the values of Curve 15. To this end the constants were determined from two points on the experimental curve — the maximum point $D = 75$, $C = 107$ cm. of cylinder, and the point $D = 16$, $C = 96$ cm. of cylinder. Whence,

$$75 = kE^2/R^2,$$

$$16 = \frac{75R^2}{R^2 + (144 \times .12)^2},$$

and

$$R = 9 \text{ ohms.}$$

With this value of R, Table III. contains the computed values of D compared with the observed values. The formula for the computation is

$$D = \frac{75}{1 + 256\left(\frac{cm}{c} - 1\right)^2}.$$

TABLE III.

C	D calc.	D obs.	C	D calc.	D obs.
53.5	.3	—	107	75	75
80.5	2.6	3.5	113	47	45
86	4.4	6.0	120	16	16.5
91	8.0	9.0	130	8	7.5
96	16	16	143	4.4	3.0
102	47	46			

The calculated values of D are plotted as the dotted curve in Fig. 7.

It is seen that the observed and the calculated values of the deflection agree well for the interval of dissonance between $C = 91$ and $C = 130$. For greater dissonance the agreement is not so good. The apparent resistance used in the calculation is 9 ohms. The actual resistance of the circuit containing the instrument was 1.40 ohms when measured with a steady current. Practically the whole of this resistance is in the coil of the instrument itself. On the supposition that Rayleigh's formula for resistance of a straight

conductor as a function of the frequency applies also to such a coil, a computation shows that the resistance of the circuit under consideration is not appreciably greater than its resistance for a steady current.

Thus the greater part of the apparent resistance of 9 ohms is to be sought as effect of the proximity of the other circuits. It is well known that the presence of a closed secondary has the effect of increasing the apparent resistance of a circuit in an amount depending on the self inductances, and resistances of the two circuits and the mutual inductance between them. When the square of the mutual inductance is negligible in comparison with the product of the two self inductances this influence of the neighboring circuit in increasing the apparent resistance is negligible. Thus for sharp resonance, as pointed out by Wien, it is necessary to make the electromagnetic connection between the circuits "loose," that is to make the square of the mutual inductance between the two circuits at each station negligible in comparison with the product of the two self inductances. The circuits employed in experiment I. did not completely fulfill this requirement, as M^2 at each station was equal to about one tenth of the product of the mutual inductances.

Experiment III. Resonance Curves with the Secondaries of 115 Turns. — Another set of resonance curves obtained with another pair of secondary coils is shown in Fig. 8. Here the secondaries had 115 turns of wire 1.57 mm. in diameter. Table IV. contains the values of the capacities corresponding to the maxima of the several curves.

TABLE IV.

Curve No.	Sending Capacity in Microfarads, S.	Optimal Rec. Cap. in Microfarads, R.	Ratio, S/R.
19	17.5×10^{-4}	4.40×10^{-4}	4.00
20	25.6	6.25	4.10
21	26.1	6.67	3.90
22	26.6	6.80	3.90
23	27.1	6.85	3.95
24	27.6	6.95	3.97
25	28.2	6.95	4.05
26	30.7	7.20	4.30
27	37.0	8.50	4.35
			Mean, 4.17

In this experiment the inductances had the following values :

	Henries.	Method of Measuring.
L_1	1.63×10^{-5}	Raleigh's bridge.
L_2 and L_3	31.0×10^{-5}	" "
L_4	6.00×10^{-5}	" " and resonance.
M_{12}	2.60×10^{-5}	Ballistic galvanometer.
M_{34}	$5.77 \times 10^{1-5}$	" "
L_1	1.44×10^{-5}	$L_4 / 4.17$, compare Table IV.

The wave-length in this case is 381 meters.

The curves of Fig. 8 were taken in the same manner as those of Fig. 6. Curve 22 when examined for percentage decrease in deflection with percentage dissonance is not quite so sharp as Curve 17 of Fig. 6. The family of curves in Fig. 8 present a different appearance from those of Fig. 6, in that the successive approximations to resonance are grouped closer around the maximum curve than they were in the former experiment. In fact in experiment III., when the dissonance between the condenser circuit and the mast at the sending station is not too great, the curves in which the sending capacity is too small (20 and 21) go up tangential to 22 on its left slope, while those in which the sending capacity is too large (23, 24, 25, and 26) come down tangential to 22 on its right slope. The maximum curve is thus a kind of envelope of the curves in its neighborhood. At first I took this to indicate persistence of the period of the condenser circuit at the receiving station, so that the progression of the maxima at the receiving station did not follow closely the progression in period at the sending station. The ratio of $S : R$ in Table IV., however, does not seem to vary in a manner to make this view tenable. So that, except for somewhat greater damping, the curves of Fig. 8 do not appear to differ essentially in character from those of Fig. 6.

I have taken other families of resonance curves with different values of the inductance and it is my intention to examine the results obtained with various relations between M^2 and the product of the self-inductances in the hope of testing some of the theoretical deductions of Wien and Abraham. Complete data for these tests are not yet at hand.

The curves here published show strikingly the necessity of tuning the condenser circuits to resonance with the masts — a necessity which has been repeatedly emphasized in the theoretical discussions of the subject, and which has also been previously shown experimentally by Mr. A. H. Taylor.

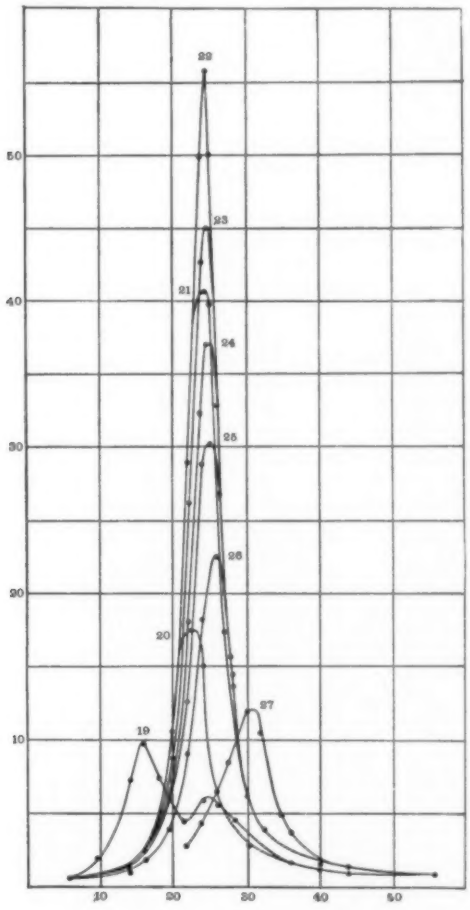

Fig. 8.

Further experiments are in progress, which should cover the case in which there is not so great a step-down in potential at the receiving station, and also the case of the directly coupled type of circuit.

IV. The Effect of Temperature on The Hewitt Mercury Interrupter.

In the course of these experiments it became early apparent that the temperature outside the bulb had considerable effect on the action of the mercury interrupter. So the interrupter was put in a thermostat consisting of a bath of oil electrically heated and with an automatic regulation of temperature. In addition to the amount of

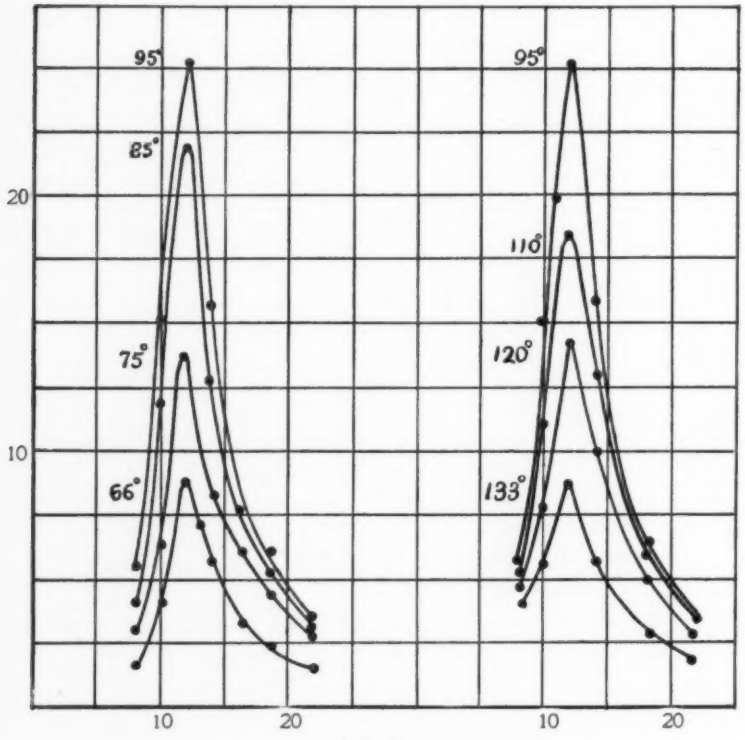

Fig. 9.

the effect, question arose whether the ohmic resistance of the interrupter depended on the temperature to such an extent as to affect the shape of the resonance curves taken at various temperatures. Fig. 9 shows the results obtained for a number of resonance curves taken with constant circuits for various temperatures of the bath surrounding the mercury interrupter. To prevent confusion the resonance curves are plotted in two groups, one for temperatures

ranging from 66° to 95° C. in which the deflections increase with increasing temperature; the other for temperatures from 95° to 133° which show a decrease of deflection with increasing temperature. The best temperature for operating this particular interrupter is 95°. At 66° and again at 133° the deflections are only one third as great as at 95°. The shape of the curves, however, examined for percentage decrease of deflection with percentage dissonance, is not materially different for the different temperatures. Of course it is not presumed that a regulation of the temperature outside the bulb results in an exact regulation of the temperature inside when the discharge is passing, but the ease with which the various observations recorded in this paper could be reproduced shows that the Hewitt interrupter, when operated in a thermostat, gives considerable uniformity in the interruption of a condenser discharge.

HARVARD UNIVERSITY,
CAMBRIDGE, MASS., June 29, 1904.

NOTES ON THE ERRORS IN THE HALF-SHADE ELLIPTICAL POLARIZER.

BY D. B. BRACE.

IN the measurements of a change of phase and of a rotation of the plane of polarization, the methods should be such that the presence of the one effect should not be confounded with that of the other. In the majority of cases the two occur separately, or can be isolated. In some instances, however, this is not possible. Thus in the differential absorption of the two circular components into which light is broken up in its passage through natural and magnetic rotary substances, plane polarized light, after its passage, is rotated and also becomes elliptically polarized. Similarly, in certain differentially absorbing double-refracting substances, the one linear component is retarded and absorbed more than the other, giving elliptical for plane polarized light, and also a rotation of the major axis. This may occur in a similar manner for plane polarized reflected and refracted light.

In the half-shade elliptical polarizer if the major axes of the vibration in the two halves are parallel and equal there will be no relative change in intensity on rotating the analyzer; if this condition is not fulfilled, there will in general be such a relative change. In the half-shade plane polarizer, if an element is interposed which is double-refracting, a change in the two halves will occur which may be appreciable, particularly if the principal plane lies between those of the polarizing elements, or if the element is not uniform in the amount of double refraction.

In the elliptical polarizer previously described[1] the discussion is based on the assumption of a crossed system of nicols with intermediate strip or element in an azimuth of 45° and a compensating section adjusted to give a match. This arrangement will in general give different azimuths to the major axes of the elliptical vibrations,

[1] PHYS. REV., February, 1904, p. 70.

PHYSICAL REVIEW, No. CIV.
(To face p. 368.) (In binding Vol. XIX. this should face p. 218.)

ERRATA.

In the article on the Errors in the Half-Shade Elliptical Polarizer, by D. B. Brace (Sept., 1904), the following corrections should be made:

In equations (34), (35), (36), (37) and (39) replace A and A' by N and N' respectively, and in the denominator of (36) replace A by N'.

Page 227, line 6 from bottom of page, read $\delta' - \delta''$ instead of $\delta' = \delta''$.

and hence, on rotation of the analyzer, the match may be destroyed. However, by lowering the order of the elements, this difference in the azimuths may be reduced so that the resulting change in intensity may not become an observable quantity. Within these limits, which are very small, a rotating section may be used; beyond this a compensating system must be used such that it may be kept crossed with that of the interposed element.

If ξ, η, A and B are the rectangular component vibrations and amplitudes respectively and ε their difference in phase, the resulting elliptical vibration is

$$\frac{\xi^2}{A^2} + \frac{\eta^2}{B^2} - \frac{2\xi\eta}{AB}\cos\varepsilon = \sin^2\varepsilon. \tag{27}$$

The angle δ between the axes of the ellipse and the coördinate axes is given by

$$\tan 2\delta = 2\frac{AB}{A^2 - B^2}\cos\varepsilon, \tag{28}$$

and if $\tan\gamma = \beta/\alpha$ is the ratio of the axes of the ellipse

$$\sin 2\gamma = \frac{2\alpha\beta}{\alpha^2 + \beta^2} = \frac{2AB}{A^2 + B^2}\sin\varepsilon. \tag{29}$$

In order to calculate these relations, the final rectangular components must be determined. Consider two such plates and let i be the azimuth of the first plate referred to the initial plane of polarization, a the azimuth of the second with respect to the first plate and s that of the analyzer with respect to the polarizer, o', e', o and e the equivalent air thickness for the ordinary and extraordinary rays in the successive plates respectively. With an initial amplitude of unity the components leaving the first plate are

$$\begin{aligned} x &= \cos i \sin 2\pi\left(\frac{t}{\tau} - \frac{o'}{\lambda}\right), \\ y &= \sin i \sin 2\pi\left(\frac{t}{\tau} - \frac{e'}{\lambda}\right), \end{aligned} \tag{30}$$

The components after passing the second plate are

$$\xi = \cos a \cos i \sin 2\pi\left(\frac{t}{\tau} - \frac{o' + o}{\lambda}\right)$$

$$+ \sin a \sin i \sin 2\pi\left(\frac{t}{\tau}-\frac{e'+o}{\lambda}\right) = A \sin 2\pi\left(\frac{t}{\tau}-\frac{C}{\lambda}\right)$$

$$\eta = -\sin a \cos i \sin 2\pi\left(\frac{t}{\tau}-\frac{o'+e}{\lambda}\right) \qquad (31)$$

$$+ \cos a \sin i \sin 2\pi\left(\frac{t}{\tau}-\frac{e'+e}{\lambda}\right) = B \sin 2\pi\left(\frac{t}{\tau}-\frac{D}{\lambda}\right),$$

where A and B are the component amplitudes and C and D the equivalent air thicknesses of a single plate which would give the same resultant vibration.

We have to determine $2AB \cos 2\pi\left(\frac{D-C}{\lambda}\right)$ and

$$2AB \sin 2\pi\left(\frac{D-C}{\lambda}\right) \quad \text{where} \quad 2\pi\left(\frac{D-C}{\lambda}\right) = \varepsilon.$$

Expanding and equating coefficients of the time function and applying the formulæ

$$AB\left(\cos 2\pi\frac{D}{\lambda}\cos 2\pi\frac{C}{\lambda} + \sin 2\pi\frac{D}{\lambda}\sin 2\pi\frac{C}{\lambda}\right) = AB \cos 2\pi\left(\frac{D-C}{\lambda}\right)$$

$$AB\left(\sin 2\pi\frac{D}{\lambda}\cos 2\pi\frac{C}{\lambda} - \cos 2\pi\frac{D}{\lambda}\sin 2\pi\frac{C}{\lambda}\right) = AB \sin 2\pi\left(\frac{D-C}{\lambda}\right)$$

we obtain a series of terms containing products of cosines and sines of the different sums of the equivalent thicknesses of the two plates taken two and two. Combining terms with like coefficients and applying the formulæ for the cosine and sine of the difference again, we have

$$2AB \cos 2\pi\left(\frac{D-C}{\lambda}\right) = -2 \sin a \cos a \cos^2 i \cos 2\pi\frac{o-e}{\lambda}$$

$$- 2 \sin^2 a \sin i \cos i \cos 2\pi\frac{(o-e)-(o'-e')}{\lambda}$$

$$+ 2 \cos^2 a \sin i \cos i \cos 2\pi\frac{(o'-e')+(o-e)}{\lambda} \qquad (32)$$

$$+ 2 \sin a \cos a \sin^2 i \cos 2\pi\frac{o-e}{\lambda}$$

$$= -\sin 2a \cos 2\pi \frac{o-e}{\lambda} - \sin^2 a \sin 2i \cos 2\pi \frac{(o-e)-(o'-e')}{\lambda}$$

$$+ \cos^2 a \sin 2i \cos 2\pi \frac{(o'-e')+(o-e)}{\lambda}$$

and

$$2AB \sin 2\pi\left(\frac{D-C}{\lambda}\right) = 2 \sin a \cos a \cos^2 i \sin 2\pi \frac{o-e}{\pi}$$

$$+ 2 \sin^2 a \sin i \cos i \sin 2\pi \frac{(o-e)+(o'-e')}{\lambda}$$

$$- 2 \cos^2 a \sin i \cos i \sin 2\pi \frac{(o'-e')+(o-e)}{\lambda} \qquad (33)$$

$$- 2 \sin a \cos a \sin^2 i \sin 2\pi \frac{o-e}{\lambda}$$

$$= \sin 2a \cos 2i \sin 2\pi \frac{o-e}{\lambda} + \sin^2 a \sin 2i \sin 2\pi \frac{(o-e)-(o'-e')}{\lambda}$$

$$- \cos^2 a \sin 2i \sin 2\pi \frac{(o'-e')+(o-e)}{\lambda}$$

As $A^2 + B^2 = 1$ we have only to determine $A^2 - B^2$. Squaring and adding terms in A and in B respectively and combining cosines and sines of the paths as before, we have

$$A^2 = \cos^2 a \cos^2 i + \sin^2 a \sin^2 i + 2 \sin a \cos a \sin i \cos i \cos 2\pi \frac{o'-e'}{\lambda}$$

$$B^2 = \sin^2 a \cos^2 i + \cos^2 a \sin^2 i - 2 \sin a \cos a \sin i \cos i \cos 2\pi \frac{o'-e'}{\lambda}.$$

Thus

$$A^2 - B^2 = \cos 2a \cos 2i + \sin 2a \sin 2i \cos 2\pi \frac{o'-e'}{\lambda}.$$

Using N', etc., the differential order for $(o'-e')/\lambda$, etc., we have

$$\tan 2\delta' = \frac{\sin 2a \cos 2i \cos 2\pi N - \sin 2i[\sin^2 a \cos 2\pi(N-N') - \cos^2 a \cos 2\pi(N+N')]}{\cos 2a \cos 2i + \sin 2a \sin 2i \cos 2\pi A'} \qquad (34)$$

and

$$\sin 2\gamma' = \sin 2a \cos 2i \sin 2\pi A + \sin^2 a \sin 2i \sin 2\pi (A - A') - \cos^2 a \sin 2i \sin 2\pi (A + A'). \qquad (35)$$

For $i = 45°$

$$\tan 2\delta' = -\frac{\cos 2a \cos 2\pi A \cos 2\pi A' + \sin 2\pi A \sin 2A'}{\sin 2a \cos 2\pi A},$$
$$= -\frac{\cos 2\pi A}{\tan 2a} + \frac{\sin 2\pi A \tan 2\pi A'}{\sin 2a}, \quad (36)$$

and

$$\sin 2\gamma' = -\cos 2a \sin 2\pi A \cos 2\pi A' - \sin 2\pi A' \cos 2\pi A. \quad (37)$$

If we take $\theta = i + a$ or refer the azimuth of the second plane to the initial plane of polarization $a = \theta - 45°$ and

$$\tan 2\delta' = \tan 2\theta \cos 2\pi N - \frac{\sin 2\pi N \tan 2\pi N'}{\cos 2\theta}, \quad (38)$$

$$\sin 2\gamma' = -\sin 2\theta \sin 2\pi N \cos 2\pi N' - \sin 2\pi A' \cos 2\pi A. \quad (39)$$

If $N' = 0$, *i. e.*, if the first plate be removed,

$$\tan 2\delta = \tan 2\theta \cos 2\pi N, \quad (40)$$

$$\sin 2\gamma = -\sin 2\theta \sin 2\pi N. \quad (41)$$

If the first plate is a double plate of orders N' and N'' where $N' > N''$, we have

$$\tan 2\delta' - \tan 2\delta'' = -\frac{\sin 2\pi N}{\cos 2\theta}(\tan 2\pi N' - \tan 2\pi N''), \quad (42)$$

or

$$\frac{\sin 2(\delta' - \delta'')}{\cos 2\delta' \cos 2\delta''} = \frac{\sin 2\pi N}{\cos 2\theta} \frac{\sin 2\pi(N' - N'')}{\cos 2\pi N' \cos 2\pi N''}, \quad (43)$$

which becomes

$$d\delta' = \frac{\cos^2 2\delta'}{\cos 2\theta} \frac{\sin 2\pi N}{2 \cos^2 2\pi N'} dN', \quad (44)$$

when $\delta' - \delta''$ and $N' - N''$ are small; and if $N'' = 0$ also, we have

$$d\delta' = -\frac{\cos^2 2\delta}{\cos 2\theta} \frac{\sin 2\pi N \tan 2\pi N'}{2}. \quad (45)$$

If $\gamma = -\gamma'$ (the minus sign must be used since the elliptical vibrations are in opposite directions), we have from (37) and (39)

$$\sin 2\theta = -\frac{\tan 2\pi\left(\frac{N'+N''}{2}\right)}{\tan 2\pi N}. \tag{46}$$

These equations will aid us in determining by inspection the angle θ for an approximate match and the inclination of the major axes to one another, together with the ratio of the axes in each vibration, from which we may determine approximately the intensity of light passing the analyzer.

Then if $N''=0$ and $N=N'=\frac{1}{90}$ and $\frac{1}{180}$ respectively, we find from (46) that we will have a match when the principal axis of the compensator N makes angles of $\theta=-14^\circ\,58'\,45''$ and $-14^\circ\,59'\,52''$, approximately, with the plane of polarization.

To the same degree of approximation we may determine $d\delta'$, the inclination to one another of the major axes of the elliptical vibrations. The exact determination of these parameters for a match and for a definite difference in intensity will require the use of the original equations (1) and (8). Thus θ determined in this way gives $-15^\circ\,0'\,36''$ and $-15^\circ\,0'\,11''$ instead of the preceding values, — differences in settings sufficient to become evident to the eye. The error is due to the unequal angles which the axes of each ellipse makes, on each side of it, with the plane of polarization as determined by (38) and (40), and, consequently, to a slight inequality of the minor axes for giving a match, a condition contrary to the equality $\gamma=-\gamma'$. The smaller the orders, however, the less this error becomes.

If in (1) $i=45$ and $N''=0$ we shall have

$$\begin{aligned} I' &= \cos^2 s - \sin 2a \sin 2(a-s)\sin^2\pi N' - \cos^2 a \cos 2(a-s)\sin^2\pi(N'+N) \\ &\quad + \sin^2 a \cos 2(a-s)\sin^2\pi(N'-N), \end{aligned}$$

and

$$I = \cos^2 s - \cos 2a \cos 2(a-s)\sin^2\pi N. \tag{47}$$

If further $N=N'$,

$$I' = \cos^2 s - \sin 2a \sin 2(a-s)\sin^2\pi N - \cos^2 a \cos 2(a-s)\sin^2 2\pi N,$$

and

$$I = \cos^2 s - \cos 2a \cos 2(a-s)\sin^2\pi N. \tag{48}$$

If now the nicols are crossed, $s = 90$ and we shall have

$$I' = \sin^2 2a \sin^2 \pi N + \cos^2 a \cos 2a \sin^2 2\pi N,$$

and

$$I = \cos^2 2a \sin^2 \pi N. \tag{49}$$

The value of a is obtained from

$$\tan^2 a = \frac{B \pm \sqrt{B^2 + 4AC}}{2A} \tag{8}$$

where, if $N'' = 0$,

$$\begin{aligned} A &= \sin \pi (2N - N'), \\ B &= 4 \sin \pi N' - \sin \pi (2N + N') + \sin \pi (2N - N'), \\ C &= \sin \pi (2N + N'), \end{aligned} \tag{8a}$$

and if further $N = N'$,

$$\begin{aligned} A &= \sin \pi N, \\ B &= 5 \sin \pi N - \sin 3\pi N, \\ C &= \sin 3\pi N. \end{aligned} \tag{8b}$$

For the more intense radiants the equivalent orders in the two portions of the field may be as low as $1/180$ and $1/360$. This would make N' $2/180$ and $2/360$ respectively, and N any value which would give for a definite angle θ the former equivalent orders.

Substituting in (8) the values of A, B and C obtained from (8*b*) on making $N = 1/90$ and $1/180$ we find

$$a = \pm 60^\circ\, 0'\, 36''$$

$$\theta = \begin{cases} -\ 15^\circ\, 0'\, 36'' \\ +\ 105^\circ\, 0'\, 36'' \end{cases}$$

and

$$a = \pm 60^\circ\, 0'\, 11''$$

$$\theta = \begin{cases} -\ 15^\circ\, 0'\, 11'' \\ \ \ 105^\circ\, 0'\, 11'' \end{cases}$$

respectively, since $\theta = a + 45^\circ$.

Using angles with the minus signs and substituting in (49) we shall find in the first case

$$I = .00030486 \quad \text{and} \quad I' = .00030487,$$

and with the second order

$$I = .000076175, \quad I' = .000076173.$$

Substituting again in (48) and making $s = 89°$ we find

$$I = .0005908, \quad I' = .0006644,$$

and hence

$$\frac{I - I'}{I} = .125 \text{ approximately,}$$

and for $N = 1/180$,

$$I = .00037611, \quad I' = .00039451$$

and

$$\frac{I - I'}{I} = .049.$$

These latter are the relative differences in intensities between the two fields when we rotate the nicol in the region most sensitive to such a change.

If now we make $N = 1/180$, $N' = 1\frac{6}{6}/180$ we find similarly

$$a = \pm 78° \; 13' \; 36'',$$
$$\theta = -33° \; 13' \; 36'',$$

and

$$I = .000255955,$$
$$I' = .000255954 \quad \text{for} \quad s = 90°,$$

and

$$I = .0005565,$$
$$I' = .0005720,$$

and

$$\frac{I - I'}{I} = .028 \quad \text{for} \quad s = 89°.$$

Comparing the results from the first and last orders, we see that for fields of like intensities the difference produced by a rotation of the nicol is between four and five times as great in the former as in

the latter. In the second example the intensity is one fourth that in the first while the change is about one third. If $N = N'/2$ the axes of the vibrations would be exactly parallel and there would be no change on rotating the nicol. The calculations show that the conditions are best met by setting the thicker section in an azimuth of 45° and so selecting the orders that the two portions of the field approach as nearly as possible a "crossed" system.

We may now calculate from (36), (37), (40), and (41) δ', δ, γ' and γ.

For $N = N' = 1/90$, $a = -60° 0' 36''$,

$$\gamma' = -59' 47''$$

$$\gamma = 1°$$

and

$$\tan\gamma' = -.017392, \quad \tan^2\gamma' = .00030248,$$

$$\tan\gamma = .017455, \quad \tan^2\gamma = .00030468,$$

$$\delta' = -15° 6' 1'',$$

$$\delta = -14° 58' 47'',$$

and

$$d\delta = 7' 14''.$$

For $N = N' = 1/180$, $a = -60° 0' 11''$.

$\gamma' = -29' 58''$, $\tan\gamma' = -.0087267$, $\tan^2\gamma' = .000076155$,

$\gamma = 30'$ $\tan\gamma = .0087268$, $\tan^2\gamma = .000076158$,

$$\delta' = -15° 1' 33'',$$

$$\delta = -14° 59' 43'',$$

and

$$d\delta = 1' 50''.$$

For

$$N = \tfrac{1}{180}, \; N = 1\tfrac{5}{6}/180, \; a = -78° 13' 26'',$$

$\gamma = -54' 59''.5$, $\tan\gamma' = -.016$, $\tan^2\gamma' = .00025601$,

$\gamma' = 55'$, $\tan\gamma = .0159979$, $\tan^2\gamma = .00025593$.

$$\delta' = -33° 14' 35'', \quad d\delta = 1' 32'',$$

$$\delta = 33° 13' 3''.$$

It is evident from the above calculations that the difference in intensity of the two portions of the field will depend upon both the inclination of the major axes of the two vibrations with one another, as well as upon the ratio of their axes in a single vibration. In the first example the major axis from the two plates makes an angle of

$$-15^\circ\,0'\,36'' - 15^\circ\,6'\,1'' = 5'\,25'',$$

and that from the single plate an angle of

$$-15^\circ\,0'\,36'' - 14^\circ\,58'\,47'' = -1'\,49'',$$

thus showing that the two ellipses are not symmetrical with respect to the plane of polarization. In the same way we find the corresponding angles in the second and third examples to be $1'\,22''$, $-28''$ and $1'\,9''$, $-23''$ respectively. Since now the ratios of the axes of each of the pairs of vibrations and consequently their squares differ by a much smaller quantity than the intensities calculated directly from (47) and (48) we conclude that this change in the match on rotating the nicol is due mainly to the inclination of the axes of the two vibrations with each other. This inclination we may examine by means of (43) and (44). Supposing it were possible to obtain a vanishing line by placing in juxtaposition and crossed two strips from the same section. If now the element to be examined is placed in the same azimuth of 45° and if $N' - N'' = 2dN = dN'$, where dN is the order of each strip, N'' will represent the order of the interposed element and N the order of the compensating plate.

Since $\delta' = \delta'' = \theta$ nearly, within the limits of a practicable system, since further from (8) $\theta_1 = \theta_2$ approximately; for two different systems in which N and N' are not too large and are in constant ratio, we see that $\delta' = \delta''$ increases approximately in proportion to N. As it has already been shown that the orders which may be used in the half-shade system itself which will show no relative change in intensity on rotating the analyzer, are so low, even with the more intense radiants, as to allow only sufficient intensity for observation, we see that a rotating compensating system N cannot be used.[1] Hence,

[1] Of course such a compensator may be used if the analyzer is first rotated through an angle equal to the rotation of the substance examined. Even when the analyzer is not disturbed, the errors will, in many instances, be small relatively to the effects observed. The great convenience of the mica rotating compensator over all others will recommend its use in very many cases.

even if the axes of the vibrations in the two portions of the field are parallel, a rotating compensator is not practicable in the half-shade type of differential refractometers. If we use a single strip dN, the same thing will hold since $2dN$ becomes dN the order of the strip. After once obtaining the sections of a half-shade which will satisfy the necessary conditions, in order to use the same, we must apply some mode of compensation in which the axes do not rotate. For small effects the strained glass strip offers the most satisfactory means. Indeed we may dispense with the second plate N and set for a match by means of the glass strip and then insert the element to be examined and reset for a match. However, for constancy in the two portions of the field under temperature and mechanical changes, a strip N' and second section N are much to be preferred.

From the third example above we see that the nearer $N = N'/2$, providing $N \lessdot N'/2$ the more nearly we approximate a true elliptical half-shade system, *i. e.*, one which will not show a differential effect when the element to be examined possesses rotary power. For very thin mica films this may be accomplished by the method of overlapping, but this will be at the expense of sharpness of the vanishing line in the field of view. Thus, suppose from the same section we overlap one portion so that the principal axes in each part are as nearly parallel to each other as possible. This double section will now serve as our strip and the other portion of the same section will, when crossed, give us the required half-shade system. These may be mounted together in this position where this is possible, or each may be cemented to its own mount. As it will be impossible to overlap such a strip with the axes exactly parallel, we may readily calculate the change in intensity for the small angle between them.

If, instead of using rectangular components, as is done in deducing Fresnel's expression (1) for the intensity of light, we use conjugate components we may reduce the number of terms and simplify the calculation. Suppose now we have such a double strip followed by the second section crossed or in subtractive order, then an element to be examined followed by another plate and the analyzer. The double strip and section are equivalent to the order of the section itself which we will designate by N_0, and a_0 its azi-

muth with the plane of polarization, N_1, N_2 those of the succeeding elements, and a_1, a_2, a' the aximuths of each element with the preceding one. Then if α and β are the conjugate components forming the resultant transmitted by the analyzer, $\alpha^2 + \beta^2 = I'$ will be the intensity of this portion of the field. The conjugate components α and β are given by the following relations:

$$\begin{aligned}\alpha = \cos \pi N_1 [&\cos \pi N_0 \cos \pi N_2 \cos (a_0 + a_1 + a_2 + a') \\ &- \sin \pi N_0 \sin \pi N_2 \cos (a_0 - a_1 - a_2 + a')] \\ - \sin \pi N_1 [&\cos \pi N_0 \sin \pi N_2 \cos (a_0 + a_1 - a_2 + a') \\ &+ \sin \pi N_0 \cos \pi N_2 \cos (a_0 - a_1 + a_2 + a')]\end{aligned} \tag{50}$$

$$\begin{aligned}\beta = \cos \pi N_1 [&\cos \pi N_0 \sin \pi N_2 \cos (a_0 + a_1 + a_2 - a') \\ &+ \sin \pi N_0 \cos \pi N_2 \cos (a_0 - a_1 - a_2 - a')] \\ + \sin \pi N_1 [&\cos \pi N_0 \cos \pi N_2 \cos (a_0 + a_1 - a_2 - a') \\ &- \sin \pi N_0 \sin \pi N_2 \cos (a_0 - a_1 + a_2 - a')].\end{aligned}$$

Make

$$a_0 = 45°, \ a_1 = 0°, \ a_2 = -75°, \ a' = 120°,$$

and

$$N_0 = \tfrac{1}{90}, \ N_1 = \tfrac{1}{6}, \ N_2 = \tfrac{1}{4}, \text{ or, } \pi N_0 = 2°, \ \pi N_1 = 30°, \ \pi N_2 = 45°.$$

This will make the polarizer and analyzer crossed.

Substituting these values in the above equations we find

$$I' = .056016.$$

If we now rotate the overlapped portion so that its principal axis makes 3° with that of the other we shall have

$$a_0 = 42°, \ a_1 = 3°, \ a_2 = -75°, \ a_1 = 120°.$$

Substituting again we have $I' = .056098$.

The change in intensity due to the rotation through 3° of this strip is thus about four times smaller than could be detected. This example is not that of a perfectly compensated or crossed system. However, the rotation of the strip destroys the compensation in part since it subtracts from the element N_1 and adds slightly to the com-

pensator N_2 and hence the intensity should increase. Such a system will, therefore, meet all the requirements of a half-shade, leaving out of consideration the comparative sharpness of the vanishing line.

If in (50) we interchange a_0 with a' and a_1 with a_2 and N_0 with N_2 we find that α and β remain unchanged and therefore that the system is a reversible one; and the intensity will not be changed if we reverse the direction of the light, or if we reverse the order of the elements themselves.

It should be noted that in the mica rotating compensator its differential dispersion is of course the same as that of the strip and once a match is obtained for any one color it will remain a match for all colors. If the differential dispersion (or its equivalent in change of phase in any case of relative retardation) of the element to be examined is not that of mica, it will not remain a match for all colors. The same will be true in using the glass-strip compensator, whose differential dispersion is probably different from that of mica as well as from that resulting from a simple change in wavelength although the latter has not yet been detected.

PHYSICAL LABORATORY,
UNIVERSITY OF NEBRASKA,
July 18, 1904.

ERRATA TO PREVIOUS PAPERS.

P. 72. line 10, read $-\dfrac{\sin\pi[2N+(N'+N'')]}{\sin\pi[2N-(N'+N'')]}$.

P. 87, line 29, read "*a gain of an order*" instead of "*a gain of half an order.*"

TESTS OF SOME OILS SUITABLE FOR USE IN AIR PUMPS.

By P. G. Nutting.

THE large and rapidly increasing use of oil flush air pumps (Fluess patent) makes it highly desirable to know what standard grades of oil available in American markets are suitable for use in these pumps. A good pump oil must have first of all a very low vapor pressure and great fluidity. The vapor pressure of the oil used of course limits the vacuum attainable. With a well cared for pump, a good Rōntgen ray vacuum should be easily and quickly attainable. Heavy, viscous oil of course makes the pump run less easily and rapidly.

From among a large number of stock samples of mineral oils kept by the Standard Oil Company, five were selected for the vapor pressure test. These were: Ruby Capital Cylinder, Dark Neutral, Standard Gas Engine, Atlantic Red, and Renown Engine. Beside these, Transformer oil supplied by the General Electric Company, and the pump oil supplied by the Pulsometer Company, were tested.

A specially constructed McLeod gauge was used in making the tests. The oil was placed in a small bulb connected with the gauge by a narrow tube, and the whole arranged to permit of a hot water bath being placed about the oil bulb and an easy and thorough cleaning between tests. The evacuating was done with a motor driven Barr & Stroud rotary mercury pump in series with Geryk oil pump.

All the oils tested give off considerable dissolved air or some light petroleum product as soon as the pressure is reduced to about 3 mm. It seems most probable that it is dissolved air that comes off. The boiling caused by it soon ceases entirely at lower pressure. When using an oil pump for very low vacua it is advisable to place the oil to be used in a bottle and exhaust it before introducing it into the pump, but for ordinary work this previous exhaustion is

unnecessary, as the pump will exhaust its own oil quite thoroughly. In fact, the vapor of volatile fluids like benzine may be pumped off with an oil pump without seriously affecting its working capacity. Chemically active gases and vapors like chlorine and bromine may also be pumped without injury to the exposed parts of the pump. Many of these gases are at once taken up by the oil, and the contaminated oil may be easily and quickly replaced by fresh.

None of these oils appears to possess a definite vapor pressure, just as they possess no definite boiling point. The pressures quoted in the table are to be regarded as superior limits to the vapor pressures rather than mean or minimum values. They are a good indication of the relative merits of the oils.

Owing to the extreme slowness with which vaporization takes place and the rapidity of action of the oil pump, a pressure much lower than the vapor pressure of the oil used may be obtained provided the leading in tubes are clean. The fact that a considerably better vacuum may be obtained when the pump is cool than when it is warm, appears to be due to the effect of temperature on the rate of vaporization rather than on the vapor pressure. With a pump using oil No. 5, whose static vapor pressure was found to be 0.015, a pressure as low as 0.004 mm. has been recorded. The vapor pressures were tested at various temperatures from zero to 60° C., but were found not to decrease more than from two to five per cent. for the whole interval.

The Transformer Oil appears to be identical with the Standard Gas Engine; while the oil originally supplied with the pump, in color and general appearance, as well as in boiling point, is very similar to the Dark Neutral.

No.	Oil.	Vapor Pressure in Millimeters of Mercury.	Name of Firm Supplying Oil.
1	Ruby capital cylinder.	0.03	Standard Oil Co.
2	Dark neutral.	0.04	" " "
3	Pump oil (No. 2).	0.02	Pulsometer Co., London.
4	Transformer oil (No. 5).	0.015	General Electric Co.
5	Standard gas engine.	0.015	Standard Oil Co.
6	Atlantic red.	0.02	" " "
7	Renown engine.	0.03	" " "

Both the Transformer and Standard Gas Engine oils showed a lower vapor pressure than the oil supplied with the pump. All three varieties are very fluid. The Standard Gas Engine oil is now being used exclusively in the pumps belonging to the bureau. It is a limpid, amber-colored petroleum product intended primarily for high temperature lubrication.

NATIONAL BUREAU OF STANDARDS.

NEW BOOKS.

The Metric Fallacy, by A. HALSEY, and *The Metric Failure in the Textile Industry*, by SAMUEL S. DALE. New York, D. Van Nostrand Company, 1904. 8vo. Pp. 231.

In the midst of the admiration universally expressed for the metric system a few voices have been raised, more noisy than numerous, to maintain that its exclusive adoption would be, if not a step backward, at least an advance of little importance, and one purchased at too great expense in many countries because of the difficult and costly period of transition. Among the authors who have undertaken to combat the metric system none has been more harsh, and, we are compelled to say, less fair than Mr. Halsey. For several years he has employed in the combat against this admirable system an amount of effort, perseverance, and vehemence worthy of a better cause, bringing forward arguments whether good or bad, and finding even in the most beautiful relations of the metric system reasons for opposing its adoption. His work is voluminous; it abounds in assertions, in citations, and in tables, whose appearance of having been duly and seriously verified might easily deceive. To criticise his argument in detail would necessitate an extended review, for there is hardly a line which does not contain a questionable statement and where it would not be easy as well as interesting to demonstrate the contrary. But we must restrict ourselves to the most typical cases, which will suffice to show in what a spirit of prejudice Mr. Halsey's work has been written.

After a short introduction the author presents the pro-metric and the anti-metric arguments. The readers of this review doubtless know them, either from having heard them many times or from their own experience. Nevertheless none of my readers would expect to see the latter argument summed up in this sentence, which will be sufficient for our purposes without quoting the rest: "We have the simplest and the most uniform system of weights and measures of any country in the world." To sign one's name to such a statement indicates at once that the writer must give up the idea of being taken seriously. Nevertheless Mr. Halsey finds difficulty in suppressing a certain admiration, even though a wholly theoretical one, for the metric system. He patriotically cites the American authors who have spoken in its favor, but he relieves himself of the need of combatting their approval by a very simple step, which consists in denying that a system which is theoretically perfect can offer any practical advantage over a system which is more complicated and of less

complete coördination. This antagonism is placed clearly in evidence in sentences such as the following: "Arguments based on 'the beautiful interrelation and correlation of the units' have little more application than a philosophical speculation regarding the appearance of the back side of the moon."

Citing also the opinion of Dr. Giding "it is simple, elastic, scientific, and on the whole a beautiful structure," Mr. Halsey replies "Was there ever such a case of rainbow chasing? Are we a nation of dreaming idealists and transcendentalists that we should be swayed by such considerations?"

Nevertheless we must not be unjust. Even while writing his sonorous sentences Mr. Halsey feels that they are purely declamatory and will not convince anybody. They are a flourish of trumpets in preparation for the attack by solid arguments, which constitute, as it were, the heavy artillery of his army. Our system, he says in substance, possesses over the metric system the inestimable advantage of being accepted without exception by all English-speaking people, while the so-called metric countries still swarm with old units, retained by custom as an imperishable evidence of the impossibility that the metric system should anywhere obtain firm foothold. The evidence for this argument is furnished by a table of about ten pages, in fine print, giving all the various nonmetric units that are still in use in countries where the law requires the use of the metric system. This would certainly be an argument of great weight if the table were not for the most part pure fantasy. Mr. Halsey however seems to treat the metric system well, for he has not cited any non-metric units employed in Denmark or in Russia; for this we should be glad to give due praise, if it were not explained by the fact that the Danish law (the only exception in Europe) does not accept the metric system, and that it has been in use in Russia for only three years. Nevertheless he cites units employed at Riga and at Copenhagen, which, if we are not mistaken, are situated in countries which he excludes from the table. The hospitality of this table is great, for it gives asylum to units of China and of Persia!

But the summit of absurdity is reached in speaking of Malta. Here, Mr. Halsey says, the barrel, containing eleven gallons, is employed as a unit. Why should the metric system be responsible for this, since Malta is an English possession? But can it be that eleven American gallons have suddenly became a British unit? It would seem difficult to believe this; and even if it were true the uniformity claimed by the author for the Anglo-Saxon units would not gain thereby, since (and Mr. Halsey cannot deny it) the English gallon is 21 per cent. larger than the American gallon. We do not reproach the Maltese for using a unit which is not contained in the English table of measures. That is of no conse-

quence to us, but it must be admitted that it is not fair to make of the fact a reproach to the metric system.

Among the units which Mr. Halsey cites there are some which no one in the country indicated by him has ever heard of. For example, one would greatly surprise the inhabitants of Neuchâtel and Geneva, two cities using the French language, by telling them that they make use of the "Stütz" or of the "Ruthe." The latter is a measure of length formerly employed in Germany, but I do not believe that the "Stütz," in spite of its German form, was ever known there. At all events as far as Neuchâtel is concerned this unit is pure invention. I have much difficulty also in believing that the "Pfundeschwer" can be a unit of length at Cairo and at Constantinople. Here, as elsewhere, Mr. Halsey has drawn his information from a bad source. He gives as reference the Special Consular Reports, Vol. XVI. Apparently this document contains typographical errors; or possibly it requires complete revision. Since Mr. Halsey makes use of official publications, how much better he might have done if, in place of reproducing a document which gives us somewhat the effect of a catalogue for a collection of antiquities, he had read and meditated upon the "Report from Her Majesties Representatives on the Metric System." But he takes care not to do this, for this document, written with an impartiality which cannot be gainsaid, is fatal for the thesis which he maintains. I doubt, however, whether he is ignorant of these "Reports," for they are mentioned among others in the replies made by a Commission of the Society of the Engineers of France to a series of questions proposed in consequence of the earlier writings of Mr. Halsey, and showing how little founded were the objections. It would have been at least polite to make reference to this latter document, which had been translated and published by the American Chamber of Commerce at Paris. Mr. Halsey does not do so, and with reason.

But Mr. Halsey's table, referred to above, does not contain merely errors; it also contains numerous truths. Unfortunately these are hard for his contention, as we shall see. Mr. Halsey tells us very truly that there is employed in Greece under the name of "Libra," and in Holland under the name of "Kan," a unit equivalent to 0.2645 gallons, and another equivalent to 2.2046 lbs.; at Milan a "Mina" of 2.6418 gallons, in Belgium a length of 3.937 inches, in Holland an "Ell" of 1.094 yards, in Germany a ton of 2204.6212 lbs. Now if he had taken the trouble to calculate the metric equivalents of these various units he would have found one liter, one kilogram, 10 liters, 10 cm., 1 meter, 1000 kilograms! Since he has given eight figures for this last unit, we may verify the fact that it is equal to the metric ton to within a gram. Mr. Halsey had decidedly bad luck in the choice of his proofs.

Relieved of its references to countries where the metric system is not legally adopted, of units that are either fantastic and unknown, or which, in spite of their local names, correspond exactly with the metric system, Mr. Halsey's table is reduced to an insignificant affair, made up of a few units which have remained in use among the common people, often only in the spoken language. Often, too, it is not a question of units at all, but rather the names of receptacles, whose capacity is specified in terms of the legal units. For example the Fass, the Eimer, and the Sac are the names of vessels for holding liquids, or substances in small pieces; their capacity has a more or less definite value determined by usage; but their contents must be measured or weighed, and are recorded in liters or kilograms. Would it not be contrary to the spirit of good law to require that wine should be sold only by the hectoliter and flour by the hundred weight?

The second part of the work written in collaboration with Mr. Dale, is of more value. It shows that in the textile industries the metric system has not been generally used and that the conditions have been met by using a number of different arbitrary units, more or less local, but with a certain preponderance of British units. We admit to Mr. Dale, as we have to Mr. Halsey, that up to the present time no convention, national or international, has fixed standard values for the pitch and diameter of screws in terms of the millimeter. These are faults which can be only gradually removed, because reforms of this nature require, in addition to national action, international agreements, which are brought about slowly. But the progress is already great. The reproaches of Messrs. Halsey and Dale would have been better justified ten years ago than now. For the textile industries international agreements, have either been made or are about to be made in several European countries. Unification on the basis of such agreements is proceeding at an increasing rate; but so radical a change in customs and habits cannot occur except at considerable expense, nor without some slight temporary loss in manufactured products. This is an objection that can be urged against any change, and it is quite comprehensible that an industrial country like the United States should have hesitated to assume such expense as long as it could be avoided. But the progressive extension of the metric system throughout the whole world leaves no doubt that the Anglo-Saxon countries will some day be forced to its adoption. Mr. Halsey claims that in countries that have adopted the metric system *two* systems are in use at the same time. If we neglect a few cases of local usage in agricultural communities, this is not true. On the other hand in the Anglo-saxon countries two systems are certainly in use, for it has been impossibe to close the door to the metric system, which the laws already recognize, and which is used exclusively by all men of science. It is this fact that

has made the old system tolerable, for one is set free from the restrictions of the old system as soon as a scientific or technical problem is approached.

C. E. GUILLAUME.

SÈVRES, FRANCE,
BUREAU INTERNATIONAL DES POIDS ET MESURES.

Le Point Critique des Corps Purs. By E. MATHIAS. 14 × 22 cm. Pp. viii + 256 Paris, C. Naud, 1904. Paper, 7 francs.

This book is an outcome of the report made in 1900 by the author before the International Congress of Physics. The opening chapter deals with the theory of Andrews, with density and saturation curves and with Natterer's tubes. Chapter II. contains a discussion of the Cagniard-Latour phenomenon and points out the insufficiency of Andrews' theory. The next three chapters are devoted to the discussion of experimental methods giving respectively three, two or one of the critical constants. In Chapter VI. the author supplements the preceding discussion with a full account of the methods for calculating the critical constants, and in the chapter following tabulates the critical data for 165 substances covering a wide range of elements and compounds.

The remainder of the book — about 60 pages — is devoted to a discussion of various theories regarding the condition of matter in the critical state. In opposition to the opinion of de Heen and others the author believes that the two-phase system liquid, vapor is univariant after equilibrium has been reached. Two classes of theories are discussed: (1) theories conforming to Avogadro's Law, and assuming the polymerization of gasogenic molecules to form liquidogenic molecules; (2) theories not conforming to Avogadro's Law, but assuming that the volume of gasogenic and liquidogenic molecules is different, or that the density of the molecule may vary. The theories of de Heen and of the author appear under the first of these heads; the theory of Traube under the second. Both de Heen and Traube ignore the pressure of the saturated vapor.

The author calls attention to the importance of time as a factor in the determination of critical phenomena, for the reason that a certain amount of time must always elapse before equilibrium between the two phases can be established. In the concluding chapter he discusses from the standpoint of the liquidogenic theory the properties of matter at temperatures above that at which the meniscus disappears, and explains the alleged existence of liquid above the critical temperature by assuming the presence of partially decomposed liquidogenic molecules.

The book is comprehensive in scope, lucid in style. That its author has himself contributed largely to our knowledge of critical phenomena is a still further reason for predicting that the book will be of interest to

all students of physical science. It is only to be regretted that in so broad a treatment of the subject the author ignores those pseudo-critical phenomena observed when superimposed liquids are heated in a closed tube. A. W. ROWE.

Festschrift, Ludwig Boltzmann gewidmet zum Sechzigsten Geburtstag 20 Februar 1904. Leipzig, Barth, 1904. Pp. xii + 930.

The invitation issued to physicists last May to contribute papers to a Jubilee Volume in celebration of the sixtieth birthday of Professor Boltzmann has resulted in the production of a book of 930 pages and containing 117 papers. On the list of authors appear eight names from the British Empire, nine from the United States, and one from Japan. The remainder of the list includes the names of 100 physicists from the continent of Europe. The book, in spite of the restriction as to space made necessary by the large number of contributors, constitutes what is probably the most representative collection of papers on physics ever gathered into a single volume. It is a worthy tribute to Ludwig Boltzmann and bears evidence of the widespread recognition which he has gained through his work. The task of editing the volume fell to Dr. Stefan Meyer to whom the hearty thanks of the contributors and physicists generally are due for the admirable promptness with which the material has been brought together and published. E. L. N.

Electrical Engineering Measurements. By G. D. ASPINALL PARR. New York, D. Van Nostrand Company, 1903. Pp. l + 322.

With scores of books dealing with the various special subjects related to electrical engineering, it seems strange that the subject of measuring instruments should not have had its share. Parr's book may be looked upon as a contribution, although it by no means covers this field. The subject is discussed under the following heads: general considerations, moving-needle electromagnetic instruments, moving-coil electromagnetic instruments, hot wire and electrostatic instruments, electromagnetic wattmeters, recording ammeters, voltmeters and wattmeters, miscellaneous standard and other instrumens, electricity supply meters. This last chapter constitutes over one third of the book.

Theory is not discussed. Many instruments are described which are of little interest to an American reader. The descriptions are general and copiously illustrated, trade cuts supplied by the manufacturer being freely used. F. B.

Die Metalle; Geschichte, Vorkommen und Gewinnung nebst ausführlicher Productions- und Preis-Statistik. Von BERNARD NEUMANN. Pp. vii + 421. Halle, Knapp, 1904.

Dr. Neumann's book on the metals, while chiefly statistical, contains,

in addition to the wealth of data concerning prices and quantities produced, a short history of each metal and an account of its occurrence and of the methods employed in preparing it for market. The volume contains information not only concerning the commoner metals such as iron, copper, lead and zinc, but likewise a brief account of the rarer metals such as vanadium, tantalium, niobium, thallium, etc. While chiefly of economic interests it will be found a valuable and interesting book of reference for the physicist and the chemist.

E. L. N.

American Meter Practice. By LYMAN C. REED. New York, McGraw Publishing Company, 1903. Pp. 1 + 196.

The scope of this book is exactly indicated by its title. The author treats the subject from the commercial as well as the technical standpoint, discussing such matters as administrative policy, the treatment of customers, methods of charging, etc. Descriptions of many forms of meters are given, together with the commercial methods of using and testing; but no discussion of theory or special refinements. The book is commended to one seeking information as to the commercial meter and its use

F. B.

Experimental Researches on Reinforced Concrete. By ARMAND CONSIDERE; translated and arranged by LEON S. MOISSEIFF. New York, McGraw Publishing Co., 1903. Pp. 1 + 188. (*Received.*)

A First Course in Infinitesimal Calculus. By DANIEL A. MURRAY. New York, Longmans, Green & Co., 1903. Pp. 1 + 432. (*Received.*)

Theoretische Grundlagen der Starkstrom-Technik. By CHARLES PROTEUS STEINMETZ. Braunschweig, Germany, Friedr. Vieweg & Sohn, 1903. Pp. 1 + 323. (*Received.*)